MAKEUP TEST - July 23rd, 2010

True/False Questions

Indicate whether the statement is true or false and explain your choice by providing a detailed answer.

Question 1.

When the price of a good rises and income remains constant, there is a substitution effect on demand but there cannot be an income effect.

Answer:
The change in demand of a good resulting from a change in the good's own price can be decomposed, using the Slutsky decomposition, into an income effects and a substitution effect. For any well behaved utility function, both of these effects exist without any additional changes in the other good's price or changes in income. In this sense, the statement above is false.

However, there exists at least one utility function, $U(x, y) = ax+by$ (perfect substitutes), for which any change in demand generated by a change in one of the good's prices will only have a substitution effect and no income effect. Another such example of no income effect is demand for good x in the case of a quasilinear utility function $U(x, y) = f(x)+y$.

(Uncertain)

Question 2.

If the equation for the demand curve is $q = a - bp$, then the ratio of marginal revenue to price is constant as price changes.

Answer:
MR can be expressed as $MR = p(1 - \frac{1}{|\varepsilon|})$, which makes the MR to price ratio equal to: $\frac{MR}{p} = 1 - \frac{1}{|\varepsilon|}$.

To see if the ratio of marginal revenue to price is constant as the price changes, we need to check if the linear demand function we are given exhibits a constant elasticity or not. For $q = a - bp$, the price elasticity of demand is given by:
$\varepsilon = \frac{p}{q}\frac{\Delta q}{\Delta p} = \frac{p}{a-bp}(-b) = \frac{-bp}{a-bp}$

Therefore, $\frac{MR}{p} = \frac{2bp-a}{bp}$ and

$\frac{\partial(MR/p)}{\partial p} = \frac{(2b)(bp)-(2bp-a)b}{(bp)^2} = \frac{2b^2p-2b^2p+ab}{(bp)^2} = \frac{ab}{(bp)^2} > 0$ for all $p > 0$.

(False)

Multiple Choice Questions

Identify the choice that best answers the question by circling it. Please justify your choice by providing an extensive answer to the question.

Question 3.

Ambrose's brother, Bartholomew, has a utility function $U(x_1, x_2) = 40\sqrt{x_1} + x_2$, where x_1 is his consumption of nuts and x_2 is his consumption of berries. His income is \$115, the price of nuts is \$5, and the price of berries is \$1. How many units of berries will Bartholomew demand?
(A) 35
(B) 16
(C) 70
(D) 22
(E) There is not enough information to determine the answer.

Answer:
Bartholomew's optimization problem is:
$max_{x_1,x_2}\ U(x_1, x_2) = 40\sqrt{x_1} + x_2$
s.t. $p_1x_1 + p_2x_2 = m$

Set up the Lagrangian:
$L(x_1, x_2, \lambda) = 40\sqrt{x_1} + x_2 - \lambda(p_1x_1 + p_2x_2 - m)$

FOC(x_1): $20x_1^{-1/2} - \lambda p_1 = 0$ (1)
FOC(x_2): $1 - \lambda p_2 = 0$ (2)
FOC(λ): $m - p_1x_1 - p_2x_2 = 0$ (3)

From (2) $\Rightarrow \lambda = 1/p_2$. Plug this in (1) to get: $20x_1^{-1/2} = p_1/p_2$, from where $x_1 = 400(p_2/p_1)^2$ and, from the budget constraint, $x_2 = (m - p_1x_1)/p_2$.

Plug in the values of p_1, p_2, and m, to get: $x_1 = 400(1/25) = 16$ and $x_2 = 115 - 80 = 35$.

Choice: A

Question 4.

The demand function for orange juice is $q = 269 - 9p$ and the supply function is $q = 9 + 4p$, where q is the number of units sold per year and p is the price per unit, expressed in dollars. The government decides to support the price of orange juice at a price floor of \$24 per unit by buying orange juice and destroying all that it has purchased. How many units must the government destroy per year?
(A) 52
(B) 56
(C) 25
(D) 61
(E) 57

Answer:
The equilibrium price is determined by the intersection of supply and demand, i.e. $269 - 9p = 9 + 4p \Rightarrow p = 260/13 = 20$. Since the price floor of 24 is above the equilibrium price, there will be excess supply in the market.

At $p = 24$, the quantity supplied is: $q_s = 9 + 4 \times 24 = 9 + 96 = 105$, and the quantity demanded is: $q_d = 269 - 9 \times 24 = 269 - 216 = 53$.

Therefore, in order to sustain a price floor of 24, the government must buy and destroy a quantity of $105 - 53 = 52$ orange juice units per year.

Choice: A

Long Answer Problems

Please provide detailed answers to each question.

Question 5.

Leo thinks leisure and consuming goods are perfect complements. Goods cost \$1 per unit. Leo wants to consume 5 units of goods per hour of leisure. Leo can work as much as he wants to at the wage rate of \$15 an hour. He has no other source of income.
(a). How many hours a day will Leo choose to spend at leisure? (2.5 points)
(b). Draw a diagram showing Leo's budget and his choice of goods and leisure. (2 points)
(c). Will Leo work more or less if his wage rate increases? Explain why. (2.5 points)

Answer:

(a) If goods and leisure are perfect complements and Leo wants to consume 5 units of goods per one hour of leisure, then his utility function is of the type: $U(C, L) = \min\{C, 5L\}$, from where: $C = 5L$ (1). Since he has no other source of income other that is labour income, Leo's budget constraint is: $1C + 15L = 24 \times 15 = 360$ (2). Substitute (1) into (2) to get: $20L = 360 \Rightarrow L = 18$

(b) Leo's consumption-leisure choice diagram, with consumption of goods on the vertical axis and leisure on the horizontal axis (that is, hours of work are measured right to left) is: - see graph.

(c) An increase in Leo's wage rate, ($w' > w$, which translates into a steeper budget line compared to the one in part(a)), will have both income (leisure enhancing) and substitution (leisure reducing) effects, but given his particular type of preferences, Leo will unambiguosly consume more leisure, and therefore work less (see the graph above). The analytical solution is as follows: since nothing alters the proportion in which Leo consumes goods and leisure, with a wage rate w' higher than w, his budget constraint becomes:

$5L' + w'L' = w'24 \Rightarrow L' = 24w'/(5 + w')$. Compare this value of optimal consumption of leisure with the one in part(a), $L = (24 \times 15)/(5 + 15)$ to get:

$\frac{24w'}{5+w'} > \frac{360}{20}$

$480w' > 1800 + 360w'$

$120w' > 1800$

which is true for any $w' > 15$ (i.e. an increase in the wage rate compared to part(a)).

Question 6.

Peter has an endowment of 3 units of good x and 5 units of good y. He can buy and sell x at a price of \$100 and y at a price of \$200. He receives an income of \$700 as alimony from a former spouse. His preferences are represented by the utility function: $U(x, y) = xy^2$.

(a). Draw Peter's budget line for x and y and explicitly show his initial endowment of x and y on your diagram. (2 points)

(b). Write an equation for Peter's budget. (1.5 points)

(c). Calculate Peter's gross demand for x and y. (2 points)

(d). Calculate Peter's net demand for x and y. (2 points)

Answer:

(a) See graph - if Peter would buy only x (i.e. horizontal intercept), $x = 2000/100 = 20$, while if he buys only y (i.e. vertical intercept), $y = 2000/200 = 10$. The slope of the budget line is $-p_x/p_y = -100/200 = -1/2$.

(b) Peter's budget line is given by (note that Peter's income is coming from his alimony and the monetary value of his endowment!):
$p_x x + p_y y = 700 + p_x \omega_x + p_y \omega_y$
$100x + 200y = 700 + 100 \times 3 + 200 \times 5$
$100x + 200y = 2000$

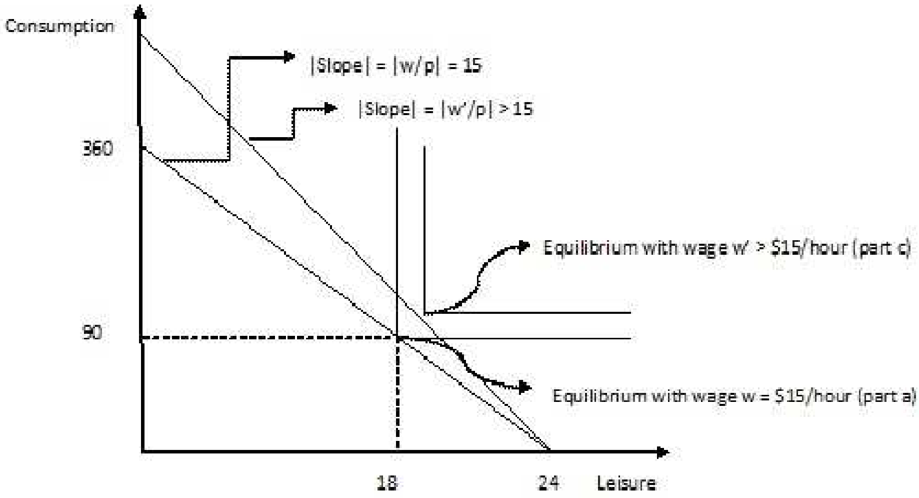
(c) Set up Peter's optimization problem:
$max_{x,y}\; U(x, y) = xy^2$
s.t. $100x + 200y = 2000$

$L(x, y, \lambda) = xy^2 - \lambda(100x + 200y - 2000)$

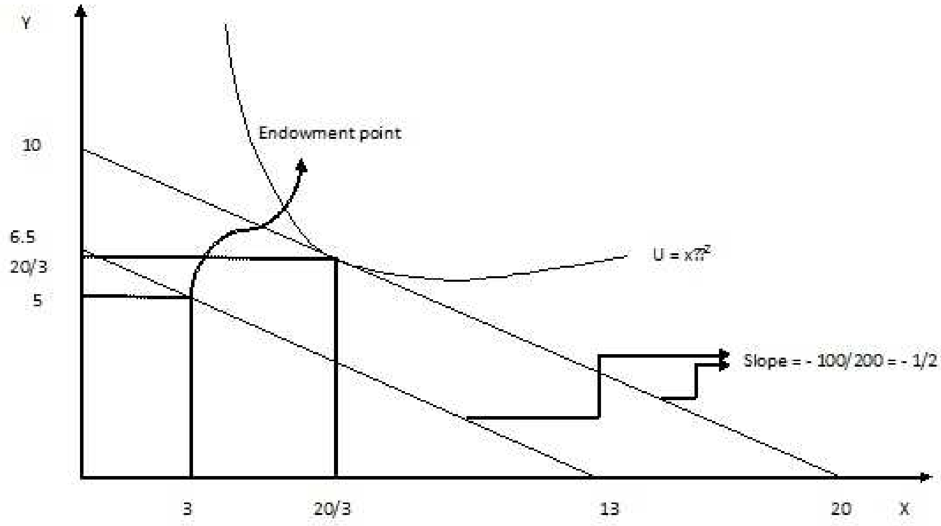
FOC(x): $y^2 - 100 = 0$ (1)
FOC(y): $2xy - 200 = 0$ (2)
FOC(λ): $2000 - 100x - 200y = 0$ (3)

From the ratio of (1) to (2) you get: $y/2x = 1/2 \Leftrightarrow x = y$. Plug this in the budget constraint to get: $300x = 2000 \Rightarrow$ Peter's gross demands (what he ends up consuming) are: $x^* = y^* = 20/3$.

(d) Peter's net demand (what he consumes less his endowment of the good) for x is $\overline{x} = x^* - \omega_x = 20/3 - 3 = 11/3$, and his net demand for y is $\overline{y} = y^* - \omega_y = 20/3 - 5 = 5/3$.


Consumption
|Slope| = |w/p| = 15
|Slope| = |w'/p| > 15
360
Equilibrium with wage w' > $15/hour (part c)
90
Equilibrium with wage w = $15/hour (part a)
18
24
Leisure

Question 5



Question 6