STAT 2507 Solution-Assignment # 2 Fall 2010

1. Use the following minitab commands to generate 8 observations:

   MTB > set c11

   DATA> 5 6 2 8 8 1 2 7

   DATA> end

   MTB > random 8 c12;

   SUBC> normal 0 0.7.

   MTB > let c1=c11+c12

   (a) [3] Use Minitab to compute the standard deviation of the numbers in c1. The answer is 3.36. (any number between 2.4 and 3.6 is acceptable)

   (b) [3] Add 2.5 to each number ('Let c2=c1+2.5' will do this, assuming the raw data are in c1). Now compute the standard deviation: 3.36. (same as part a) How does your answer compare with that in part (a)? Same.

   (c) [3] Multiply the data in (a) by 4 and compute the standard deviation: 13.44. (part a × 4) How does your answer compare with that in part (a)? 4 times. (If you are not sure, divide the standard deviation of part (c) by the one of part (a).)

2. Suppose that X has a binomial distribution with n=30 and p=0.6. Use minitab to simulate 50 values of X.

   MTB > random 50 c1;

   SUBC > binomial 30 0.6.
   (a) [5] What proportion of your values are less than 20? 36/50. (any number between 25/50 and 40/50 is acceptable)

   (b) [5] What is the exact probability that X will be less than 20? 0.70850.
   Hint: To find $P(X \leq k)$, for any $k \geq 0$, use the 'cdf' command; this works by typing

   MTB > cdf;

   SUBC > binomial 30 0.6.

   (c) [6] Find $P(X < 24)$= 0.9828, and $P(9 \leq X \leq 15)$= 0.1755-0.0002=0.1753 .

   (d) [5] If you simulate 10000 values of X, what would be the expected number of values (among the 10000) that are less than 24? (1000)(0.9828)=9828.

3. Suppose that X has a Poisson distribution with mean μ=15. Use the 'cdf' command
   MTB > cdf;
   SUBC > poisson 15.
   Find $P(X < 10)$= [3]0.06985, and $P(15 \leq X \leq 20)$= [3] 0.9170-0.4657=0.4513.

ALSO do the following questions:

1. In a certain factory, Machines A, Machine B, and Machine C are all producing widgets. Widgets produced by Machine A have a 1% chance of being defective. Likewise, widgets produced by Machine B and Machine C are defective 4% and 2% of the time, respectively. Of the total production of widgets in the factory, Machine A produces 30%, Machine B produces 25%, and Machine C produces 45%. Suppose a widget is selected at random from this factory.
(a) [8] What is the probability the widget is defective?
Solution:
D: defective widgets, A: Machine A, B: Machine B, C: Machine C, so

$$P(D|A) = 0.01, P(D|B) = 0.04, P(D|C) = 0.02, P(A) = 0.3, P(B) = 0.25, P(C) = 0.45$$

$$P(D) = P(D|A)P(A)+P(D|B)P(B)+P(D|C)P(C) = 0.01(0.3)+0.04(0.25)+0.02(0.45) = 0.022$$

(b) [8] If the widget is defective, what is the probability it was produced by Machine B?
Solution:

$$P(B|D) = \frac{P(D|B)P(B)}{P(D)} = \frac{0.04(0.25)}{0.022} = 0.45$$

2. A student is preparing for an upcoming exam. The professor for the course has given the class 30 questions to study from and plans to select 10 of the questions for use on the actual exam. Suppose that the student knows how to solve 25 of the 30 questions.
(a) [7] What is the probability that the student will get perfect on the test?
Solution:
X = number of correct answers has an hypergeometric distribution where $N = 30,\ n = 10,\ M = 25$, so

$$P(X = 10) = \frac{C_{10}^{25} C_0^5}{C_{10}^{30}} = 0.109$$

(b) [7] What is the probability that the student will get at least 8 questions correct on the test?
Solution:

$$P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10) = \frac{C_8^{25} C_2^5}{C_{10}^{30}} + \frac{C_9^{25} C_1^5}{C_{10}^{30}} + \frac{C_{10}^{25} C_0^5}{C_{10}^{30}} = 0.809$$

3. The number of flaws on a VHS magnetic tape produced continuously at a factory follows a Poisson distribution with an average of 0.01 flaws per meter. A standard VHS cassette tape contains 250 meters of magnetic tape.
(a) [7] What is the probability that there are at least two flaws in a single VHS cassette tape?
Solution:
X=number of flaws is a Poisson random variable with$\mu = 0.01$ per meter, or $\mu = 0.01(250) = 2.5$ per cassette tape

$$P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\frac{e^{-2.5}(2.5)^0}{0!} + \frac{e^{-2.5}(2.5)^1}{1!}\right] = 0.7127$$

(b) [7] What is the probability that there are no flaws in a single VHS cassette tape; that is, a tape is flawless?
Solution:

$$P(X = 0) = \frac{e^{-2.5}(2.5)^0}{0!} = 0.082$$

(c) [8] In a random sample of 20 cassettes, what is the probability that at least two of them are flawless?

Solution:

Y = number of flawless cassette has Binomial distribution with $n = 20,\ p = 0.08$, so

$$P(Y \geq 2) = 1 - \Big[P(X = 0) + P(X = 1)\Big] = 1 - \left[\binom{20}{0}(0.08)^0(0.92)^{20} + \binom{20}{1}(0.08)^1(0.92)^{19}\right] = 0.483$$

4. For events A and B we have

$$P(A) = 0.3 \qquad P(B) = 0.8 \qquad P(A \cup B) = 0.9$$

(a) [9] Find $P(A|B) \quad P(A' \cap B) \quad P(B' \cup A')$.

Solution:

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow P(A \cap B) = 0.3 + 0.8 - 0.9 = 0.2$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.8} = 0.25$

$P(A' \cap B) = P(B) - P(A \cap B) = 0.8 - 0.2 = 0.6$

$(B' \cup A') = 1 - P(A \cap B) = 1 - 0.2 = 0.8$

(b) [3] Are A and B independent? Why?

Solution:

No there are not independent, since

$P(A \cap B) \neq P(A)P(B), 0.2 \neq (0.3)(0.8)$