Econ 301
Problem Set #1
and answers

There are three questions. The problem set is due October 17, 17:00. Please put your assignments in my departmental mail box.

1. A consumer has a utility function $u(x_1, x_2) = a_1x_1 + a_2x_2 + x_1x_2$ and a budget constraint $p_1x_1 + p_2x_2 = m$. Are goods 1 and 2 normal or inferior for this person? Compute the price and income elasticities at the point $(p_1, p_2, m) = (1, 1, 10)$.

2. A consumer has a utility function

$$u(x_1, x_2) = a_1 \ln(x_1 - a_1) + \ln(x_2 - a_2)$$

Derive the Marshallian and Hicksian demand functions for this consumer and show that they are downward sloping in their own prices. Derive the indirect utility function and show that it is homogeneous of degree zero in its arguments.

3. For a consumer with a utility function $u(x_1, x_2) = a_1 \ln(x_1) + a_2 \ln(x_2)$ and the same budget constraint as in question 1 compute the welfare loss to this consumer of a change in p_2 to $2p_2$ both in terms of utility and money.

Answers

1. The first order condition for constrained utility maximization is

$$\frac{a_1 + x_2}{a_2 + x_1} = p_1/p_2$$

when this is combined with the budget constraint the demand functions become

$$\begin{aligned} x_1(p_1, p_2, m) &= \frac{m - a_2p_1 + a_1p_2}{2p_1} \\ x_2(p_1, p_2, m) &= \frac{m - a_1p_2 + a_2p_1}{2p_2} \end{aligned}$$

The income and price elasticities are

$$\begin{aligned} \frac{m\partial x_1}{x_1\partial m} &= \frac{m}{m - p_1a_2 + a_1p_2} = \frac{10}{10 + a_1 - a_2} \\ \frac{p_1\partial x_1}{x_1\partial p_1} &= \frac{-a_2p_1}{m - a_2 + a_1p_2} - 1 = \frac{-a_2}{10 + a_1 - a_2} - 1 \end{aligned}$$

etc. Both goods are normal and since the cross price elasticities are positive the goods are substitutes.

2. Here the first order condition is

$$\frac{a_1/(x_1 - a_1)}{1/(x_2 - a_2)} = p_1/p_2$$

this and the budget constraint lead to

$$
\begin{aligned}
x_1(p_1, p_2, m) &= \frac{(m - a_2 p_2 + p_1)a_1}{(a_1 + 1)p_1} \\
x_2(p_1, p_2, m) &= \frac{m - a_1 p_1 + a_2 p_2}{(a_1 + 1)p_2}
\end{aligned}
$$

as demand functions which are downward sloping in their own prices since $m - a_2 p_2$ and $m - a_1 p_1$are both positive.

The Hicksian or compensated demand functions use the first order condition and the utility constraint

$$a_1 \ln(h_1 - a_1) + \ln(h_2 - a_2) = u^0$$

take natural logarithms of the first order condition to get

$$
\begin{aligned}
\ln(h_2 - a_2) &= \ln(h_1 - a_1) + \ln(p_1/a_1 p_2) \\
&= \ln(h_1 - a_1) + q
\end{aligned}
$$

and use the utility constraint to get

$$
\begin{aligned}
h_1(p_1, p_2, u^0) &= a_1 + \exp[(u^0 - q)/(a_1 + 1)] \\
h_2(p_1, p_2, u^0) &= a_2 + \exp[(u^0 - a_1 q)/(a_1 + 1)]
\end{aligned}
$$

These two functions are downward slopping in their own prices.

The indirect utility function

$$
\begin{aligned}
V(p_1, p_2, m) &= a_1 \ln[\frac{(m - a_2 p_2 + p_1)a_1}{(a_1 + 1)p_1}] + \\
& \ln[\frac{m - a_1 p_1 + a_2 p_2}{(a_1 + 1)p_2}]
\end{aligned}
$$

is homogeneous of degree zero because the demand functions are.

3. This utility function leads to

$$
\begin{aligned}
x_1(p_1, p_2, m) &= \frac{a_1 m}{(a_1 + a_2)p_1} \\
x_2(p_1, p_2, m) &= \frac{a_2 m}{(a_1 + a_2)p_2}
\end{aligned}
$$

it has an indirect utility function

$$V(p_1, p_2, m) = a_1 \ln[\frac{a_1 m}{(a_1 + a_2)p_1}] + a_2 \ln[\frac{a_2 m}{(a_1 + a_2)p_2}]$$

The loss in terms of utility when p_2 increases to $2p_2$ is

$$V(p_1, p_2, m) - V(p_1, 2p_2, m) = -a_2[\ln(2)]$$

the loss in terms of money is just compensating variation which is the value of c which satisfies the equation

$$a_1 \ln(m) + a_2 \ln(m) = a_1 \ln(m + c) + a_2 \ln(m + c) - \ln(2)$$

or

$$c = m(2^{a_1 + a_2} - 1)$$