CARLETON UNIVERSITY

FINAL EXAMINATION
APRIL 2007

DURATION: 3 HOURS

Department Name and Course Number: Mathematics and Statistics, MATH 1005
Course Instructor(s): Dr. A. Alaca, Dr. S. Melkonian, Dr. R. Krechetnikov

AUTHORIZED MEMORANDA
Non-programmable, non-graphing calculator permitted.

[4] 1. The orthogonal trajectories of the family of curves $y = kx^3$ satisfy the differential equation

(a) $y' = \dfrac{3y}{x}$ (b) $y' = -\dfrac{3y}{x}$ (c) $y' = \dfrac{x}{3y}$ (d) $y' = -\dfrac{x}{3y}$ (e) None of these

[4] 2. The solution y of the initial-value problem $y' = \dfrac{x^2}{2y}$, $y(0) = -4$, satisfies $y(3) =$

(a) -7 (b) 7 (c) 5 (d) -5 (e) None of these

[4] 3. A particular solution of the equation $y'' - 4y' + 4y = 3e^{2x}$ is given by $y_p =$

(a) Ce^{2x} (b) Cxe^{2x} (c) Cx^2e^{2x} (d) Cx^3e^{2x} (e) None of these

[4] 4. The general solution of the equation $x^2y'' + 5xy' + 13y = 0$ for $x > 0$ is given by $y =$

(a) $e^{-2x}[c_1 \cos(3\ln x) + c_2 \sin(3\ln x)]$
(b) $e^{2x}[c_1 \cos(3\ln x) + c_2 \sin(3\ln x)]$
(c) $c_1 \cos(2x) + c_2 \sin(2x)$
(d) $c_1 \cos(2\ln x) + c_2 \sin(3\ln x)$
(e) None of the above

[4] 5. The sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}}{4 \cdot 3^n}$ is

(a) $\dfrac{3}{2}$ (b) 3 (c) $\dfrac{2}{3}$ (d) 2 (e) None of these

[4] 6. The radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{2^n(n+1)} x^n$ is

(a) $\dfrac{3}{2}$ (b) ∞ (c) $\dfrac{2}{3}$ (d) 0 (e) None of these

[4] 7. The interval of convergence of the series $\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$ is

(a) $(-2,2)$ (b) $(1,5)$ (c) $[1,5)$ (d) $(1,5]$ (e) None of these

[4] 8. The power-series representation of the function $f(x) = \frac{x}{1+x^2}$ about the point $a=0$ is

(a) $\sum_{n=0}^{\infty} x^{2n+1}$ (b) $\sum_{n=0}^{\infty}(-x)^{2n+1}$ (c) $\sum_{n=0}^{\infty}(-1)^n x^{2n+1}$ (d) $\sum_{n=0}^{\infty}(-1)^n x^{2n}$

(e) None of these

[4] 9. The coefficient of x^3 in the Taylor series of the function $f(x) = \sqrt{1+x}$ about the point $a=0$ is

(a) $-\frac{1}{16}$ (b) $\frac{3}{8}$ (c) $\frac{1}{16}$ (d) $-\frac{3}{8}$ (e) None of these

[4] 10. Given the series (i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ and (ii) $\sum_{n=0}^{\infty} \frac{2^{n+3}}{3^n}$,

(a) (i) converges conditionally and (ii) converges absolutely
(b) Both (i) and (ii) converge absolutely
(c) (i) converges conditionally and (ii) diverges
(d) Both (i) and (ii) diverge
(e) None of the above

[4] 11. (a) Find the general solution of the equation $y' + 2y = e^{-2x}\cos(x)$.

[6] (b) Solve the initial-value problem $y' = \frac{x^2+2y^2}{2xy}$, $y(1) = 2$.

[5] 12. (a) Find the general solution of the equation $3x^2y + y + y^4 + (x + x^3 + 4xy^3 + 3y^2)y' = 0$.

[5] (b) Show that the differential equation $x + y^2 - 2yy' = 0$ is not exact and find an integrating factor which makes it exact. Write down the new exact differential equation, but do not solve it.

[4] 13. (a) Find two linearly independent solutions of the equation $y'' + 4y' + 4y = 0$.

[6] (b) Find the general solution of the equation $y'' + 4y' + 4y = \frac{1}{xe^{2x}}$, $x > 0$.

[12] 14. For each one of the following series, determine whether it converges absolutely, converges conditionally, or diverges. Justify your answer.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{\sqrt{n^8+3n+1}}$ (b) $\sum_{n=1}^{\infty} \frac{2^n(n+1)}{n \cdot 3^n}$ (c) $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln(n)}$

[5] 15. (a) Find the Taylor series of the function $f(x) = e^{2x}$ about the point $a=1$.

[5] (b) Find the Taylor series of the function $f(x) = \ln(x)$ about the point $a=1$.

[8] 16. Solve the initial-value problem $y'' - xy' - 2y = 0$, $y(0) = 0$, $y'(0) = 1$.

Answers to the multiple choice questions

1. (d)	6. (c)
2. (d)	7. (b)
3. (c)	8. (c)
4. (e)	9. (c)
5. (a)	10.(b)

Answers to the long answer questions

11(a) $y = e^{-2x}(c + \sin x)$.

11(b) $x = e^{-4}e^{y^2/x^2}$.

12(a) $x^3y + xy + xy^4 + y^3 = C$.

12(b) $I(x) = e^{-x}$, New equation $e^{-x}(x + y^2) - 2e^{-x}yy' = 0$.

13(a) $y_1 = e^{-2x}$, $y_2 = xe^{-2x}$.

13(b) $y = c_1e^{-2x} + c_2xe^{-2x} + xe^{-2x}(-1 + \ln x)$.

14. (a) absolutely convergent, (b) absolutely convergent, (c) conditionally convergent.

15. $e^{2x} = \sum_{n=0}^{\infty} \frac{e^2 2^n (x-1)^n}{n!}, \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}(x-1)^n}{n}.$

16. $y = x \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = x \sum_{n=0}^{\infty} \frac{(\frac{x^2}{2})^n}{n!} = xe^{x^2/2}.$