

Assignment 1:

KINEMATICS 1-D Motion

Assigned: Sept 5 14:30 Due: September 13 19:00

1 A ferry boat transports tourists among three islands. It sails from the first island to the second island, 4.76 km away, in a direction  $37.0^\circ$  north of east. It then sails from the second island to the third island in a direction  $69.0^\circ$  west of north. Finally it returns to the first island, sailing in a direction  $28.0^\circ$  east of south. Calculate the distance between (a) the second and third islands (2p) (b) the first and third islands. (2p)

Let  $A$  represent the distance from island 2 to island 3. The displacement is  $\mathbf{A} = A$  at  $159^\circ$ . Represent the displacement from 3 to 1 as  $\mathbf{B} = B$  at  $298^\circ$ . We have 4.76 km at  $37^\circ + \mathbf{A} + \mathbf{B} = \mathbf{0}$ .

$$(4.76 \text{ km}) \cos 37^\circ + A \cos 159^\circ + B \cos 298^\circ = 0$$

For x-component  $3.80 \text{ km} - 0.934A + 0.469B = 0$

$$B = -8.10 \text{ km} + 1.99A$$

For y-components,

$$(4.76 \text{ km}) \sin 37^\circ + A \sin 159^\circ + B \sin 298^\circ = 0$$

$$2.86 \text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating  $B$  by substitution:

$$2.86 \text{ km} + 0.358A - 0.883(-8.10 \text{ km} + 1.99A) = 0$$

$$2.86 \text{ km} + 0.358A + 7.15 \text{ km} - 1.76A = 0$$

$$10.0 \text{ km} = 1.40A$$

$$A = \boxed{7.17 \text{ km}}$$

(b)  $B = -8.10 \text{ km} + 1.99(7.17 \text{ km}) = \boxed{6.15 \text{ km}}$

2 A particle initially located at the origin has an acceleration of  $\vec{a} = 3\vec{j} \text{ (m/s}^2\text{)}$  and an initial velocity of  $\vec{v}_i = 5\vec{i} \text{ (m/s)}$

.Find (a) the vector position and velocity vector at any time  $t$  and (b) the coordinates of position vector at  $t=2\text{s}$  (4p)

$$\mathbf{a} = 3.00\hat{j} \text{ m/s}^2; \mathbf{v}_i = 5.00\hat{i} \text{ m/s}; \mathbf{r}_i = 0\hat{i} + 0\hat{j}$$

(a)  $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2} \mathbf{a} t^2 = \left[ 5.00t\hat{i} + \frac{1}{2} 3.00t^2\hat{j} \right] \text{ m}$   $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a} t = \left( 5.00\hat{i} + 3.00t\hat{j} \right) \text{ m/s}$

(b)  $t = 2.00 \text{ s}$ ,  $\mathbf{r}_f = 5.00(2.00)\hat{i} + \frac{1}{2}(3.00)(2.00)^2\hat{j} = (10.0\hat{i} + 6.00\hat{j}) \text{ m}$

$$\text{so } x_f = \boxed{10.0 \text{ m}}, y_f = \boxed{6.00 \text{ m}}$$

$$\mathbf{v}_f = 5.00\hat{i} + 3.00(2.00)\hat{j} = (5.00\hat{i} + 6.00\hat{j}) \text{ m/s}$$

$$v_f = |\mathbf{v}_f| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81 \text{ m/s}}$$

3 A rock is dropped from rest into a well. The sound of the splash is heard 2.40 s after the rock is released from rest. (a) How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (3p) (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated? (1p)

SOLUTION:

(a)  $d = \frac{1}{2}(9.80)t_1^2$ ;  $d = 336t_2$ ;  $t_1 + t_2 = 2.40$   $336t_2 = 4.90(2.40 - t_2)^2$

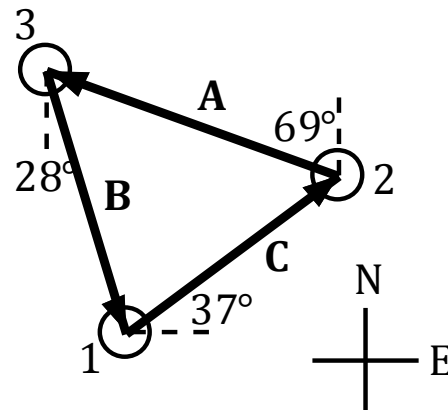
$$4.90t_2^2 - 359.5t_2 + 28.22 = 0; \text{ so that } t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$$

$$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765 \text{ s} \text{ so } d = 336t_2 = \boxed{26.4 \text{ m}}$$

(b) Ignoring the sound travel time,  $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2 \text{ m}$ , an error of.  $\boxed{6.82\%}$

$$\text{a) } d = 26.4 \text{ m}$$

$$\text{b) } 6.82\%$$



4. The height of a helicopter above the ground is given by  $h = 3.00t^3$ , where  $h$  is in meters and  $t$  is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground? (4p)

SOLUTION:

$$y = 3.00t^3: \text{ At } t = 2.00 \text{ s}, y = 3.00(2.00)^3 = 24.0 \text{ m and}$$

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s } \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting  $y_b = 0$ ,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for  $t$ , (only positive values of  $t$  count),  $t = 7.96 \text{ s}$ .

5 Speedy Sue, driving at 30.0 m/s, enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can accelerate only at  $-2.00 \text{ m/s}^2$  because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van. (4p)

Take the original point to be when Sue notices the van. Choose the origin of the  $x$ -axis at Sue's car. For her we have  $x_{is} = 0$ ,

$v_{is} = 30.0 \text{ m/s}$ ,  $a_s = -2.00 \text{ m/s}^2$  so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)t^2.$$

For the van,  $x_{iv} = 155 \text{ m}$ ,  $v_{iv} = 5.00 \text{ m/s}$ ,  $a_v = 0$  and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00 \text{ m/s})t + 0.$$

To test for a collision, we look for an instant  $t_c$  when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$

$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } 11.4 \text{ s}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155 \text{ m} + (5.00 \text{ m/s})(11.4 \text{ s}) = 212 \text{ m}.$$

Collision  
takes place  
at 212m