Solutions to review problems for the mid term:

Problem 1:

a. First we estimate the demand curve

$Q = a_0 - b_0P$

$b_0 = 25$

$Q = a_0 - b_0P$

$1206 = a_0 - 25(41)$

$1206 = a_0 - 1025$

$a_0 = 2231$

$Q_0 = 2231 - 25P$

Next, we estimate the supply curve

$Q = a_1 + b_1P$

$b_1 = 50$

$Q = a_1 + b_1P$

$1206 = a_1 + 50(41)$

$a_1 = -844$

$Qs = -844 + 50P$

b. Multiply demand equation by 1.10

$1.10\,(2231 - 25P)$

$Qd' = Qs$ and solve

$Qs = -844 + 50P$

Set $Qd' = Qs$ and solve.

$2454.1 - 27.5P = -844 + 50P$

$3298.1 = 77.5P$

$P = 42.56$

Substitute P into Qd' to find quantity demanded

$Qd' = 2454.1 - 27.5(42.56)$

$Qd' = 1283.7$ or 1284

c. Since price cannot rise, the shortage will be the quantity demanded with the new demand minus the quantity supplied with the unchanged supply

Quantity demanded: $Q = 2454.1 - 27.5(41) = 1326.6$

Quantity supplied: $Q = -844 + 50(41) = 1206.0$

Shortage $= 1326.6 - 1206.0 = 120.6$ tons per week.

Problem 2:

1. The total cost is defined as Cost=FC+VC

$FC=4*100=400$

We substitute the fixed capital into the production function equation: $q=L^{0.5}(100)^{0.5} \Leftrightarrow L=q^2/100$

Hence,

$VC=2/100\ q^2$

$TC=400+0.02\ q^2$

2. The first derivative function of the total cost function is: $MC=0.04q$

3. The minimum cost is achieved at: MC=ATC

$0.04q=400/q+0.02q \Leftrightarrow q=141.42$

4. The labor needed to achieve the lowest level of cost in the short tern is: $L=141.42^2/100=200$

5. The optimal ratio in the long run is: $MRTS=w/r \Leftrightarrow K/L=w/r=2/4$

   $K=0.5L$

6. $500=L^{0.5}\,(0.5L)^{0.5}=0.5^{0.5}L$

$L=500/0.5^{0.5}=707.10$
$K=0.5*707.10=353.55$
$Cost=707.10*2+353.55*4=2828.4$

Problem 3:

ⓐ

$$MP_L = 0.25 \times 0.8 \times K^{0.2} L^{-0.2}$$
$$MP_K = 0.25 \times 0.2 \times K^{-0.8} L^{0.8}$$

$$\frac{MP_L}{MP_K} = \frac{0.25 \times 0.8 \times K^{0.2} L^{-0.2}}{0.25 \times 0.2 \times K^{-0.8} L^{0.8}} = 4\frac{K}{L}$$

$$\frac{MP_L}{MP_K} = \frac{w}{r}$$
$$4\frac{K}{L} = \frac{12}{48}$$
$$L = 16K$$

ⓑ

$$200{,}000 = 12L + 48K \iff 200{,}000 = 12(16K) + 48K$$
$$200{,}000 = 240K \iff K = 833.33$$

$$L = 16 \times 833.33 = 13\,333.33$$
$$q = 0.25(833.33)^{0.2}(13333.33)^{0.8} = 1914$$

ⓒ New output $q' = 1914 + 200 = 2114$

$$2114 = 0.25(833.33)^{0.2} L^{0.8}$$
$$L = \left(\frac{2114}{0.25(833.33)^{0.2}}\right)^{1/0.8} = 15092.17$$

ⓓ

$$Cost = 48 \times 833.33 + 12 \times 15092.17 = 221105.99$$

ⓔ Check the graph in the book, page: 253.