York University

Faculty of Science and Engineering

Math 1190 M

Class Test 1

NAME (print):____

(Family) (Given)

SIGNATURE:____

STUDENT NUMBER:____

ANSWERS

Instructions:

1. Time allowed: 50 minutes.
2. **NO CALCULATORS OR OTHER AIDS PERMITTED**
3. Show your work. Your work must justify any answers you give. Use page backs for any scrap work.
4. Use pen to fill in cover. If you use pencil for your solutions, you may not submit your paper for regrading.
5. There are 7 questions on 6 pages.

Question	Points	Marks
1	20	
2	20	
3	20	
4	10	
5	15	
6	10	
7	5	
Total	100	

1. (20 points) Establish which of the following are tautologies. Use truth tables to justify your answer.

   (a) $p \wedge q \;\rightarrow\; (p \rightarrow q)$.

This is a tautology

p	q	$p \wedge q$	$\rightarrow$	$(p \rightarrow q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	T
F	F	F	T	T

   (b) $\neg p \wedge (p \rightarrow q) \;\rightarrow\; \neg q$. This is not a tautology

p	q	$\neg p$	$\wedge$	$(p \rightarrow q)$	$\rightarrow$	$\neg q$
T	T	F	F	T	T	F
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

2. (20 points) Establish which of these pairs are logically equivalent. Use truth tables to justify your answer.

(a) $(p \vee q) \wedge r$ and $p \vee (q \wedge r)$.　　These are NOT logically equivalent.

p	q	r	$(p \vee q) \wedge r$			$p \vee (q \wedge r)$		
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T	F
T	F	T	T	T	T	T	T	F
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F	F
F	F	T	F	F	T	F	F	F
F	F	F	F	F	F	F	F	F

(b) $p \vee (p \wedge q)$ and $p \wedge (p \vee q)$.　　These are logically equivalent.

p	q	$p \vee (p \wedge q)$			$p \wedge (p \vee q)$		
T	T	T	T	T	T	T	T
T	F	T	T	F	T	T	T
F	T	F	F	F	F	F	T
F	F	F	F	F	F	F	F

3. (20 points) Consider the statement,

If 3 is an even number then $3(3) + 1$ is an even number.

(a) Is the statement true? Justify your answer.

Let p be 3 is even. p is F.
Let q be $3(3)+1$ is even. q is T

$F \rightarrow T$ is true!

The statement is true!

(b) State its contrapositive. Is its contrapositive true? Justify your answer.

If $(3)+1$ is odd (or NOT EVEN)
then 3 is odd.

$F \rightarrow T$ is True.

The contrapositive of the statement is true! Also OKAY to simply indicate that statement and its contrapositive have the same truth values.

(c) State its converse. Is its converse true? Justify your answer.

If $3(3)+1$ is even then 3 is even

$T \rightarrow F$ is false.

(d) State its negation. Is its negation true? Justify your answer.

~~[illegible]~~

3 is even and $3(3)+1$ is odd

$F \wedge F$ is false

OR The negation of a true statement is false.

4. (10 points) If $p \to q$ is known to be true, which of the following must necessarily be true as well. Briefly justify your answer.

(a) $q \to p$.

This is the converse. Their truth tables are different:

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

So $q \to p$ can be false when $p \to q$ is true.

(b) $\neg q \to \neg p$.

This is the contrapositive. Their truth tables are the same:

p	q	$p \to q$	$\neg q$	$\neg q \to \neg p$	$\neg p$
T	T	T	F	T	F
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

So whenever $p \to q$ is true $\neg q \to \neg p$ must necessarily be true.

5. (15 points) Let $P(x, y)$ be the statement, "$x^2 - y = 0$. If the universe of discourse consists of all integers (negative, zero and positive whole numbers), what are the truth values of each of the following. Briefly justify your answers.

(a) $P(4, 2)$.

$4^2 - 2 = 0$ is false

(b) $P(2, 4)$.

$2^2 - 4 = 0$ is true

(c) $\exists x\, P(x, -9)$.

This says $x^2 + 9 = 0$ has an integer as solution. This is false.

(d) $\exists x \exists y\, P(x, y)$.

This is true. Just (as an instance) ~~Det~~ check $x=1$ $y=1$.

(e) $\forall x \exists y\, P(x, y)$.

This is true. For $x=a$ let $y=a^2$.

(f) $\forall y \exists x\, P(x, y)$.

This is false. Let $y=-1$ (as an instance). Then $x^2+1=0$ has no solutions.

(g) $\forall x \forall y\, P(x, y)$.

This is false. Let $x=0$, $y=1$ (as an instance). $0^2 - 1 = 0$ is false.

6. (10 points) Express each of the following so that all negation symbols immediately precede predicates.

(a) $\neg(\ \forall x \exists y\, P(x,y)\ \vee\ \exists y\, Q(y)\ )$.

$$\exists x \forall y \neg P(x) \wedge \forall y \neg Qy$$

(b) $\neg(\ \exists x(R(x) \to Q(x))\ )$.

$$\forall x \neg (R(x) \to Q(x))$$

$$\equiv \forall x (R(x) \wedge \neg Q(x))$$

This parentheses are necessary.

7. (5 points) Consider the statement $(P(3) \to Q(3)) \ \to\ (\ \exists x\, P(x)\ \to \exists x\, Q(x)\ )$ with universe of discourse the set of integers. Show that it is **not** a tautology by choosing P and Q to make the statement false.

Want $P(3) \to Q(3)$ true

with $\exists x P(x) \to \exists x Q(x)$ false

Make $P(3)$ false, $Q(x)$ always false

Example:

$P(x)$: x is even

$Q(x)$: $x^2 < 0$.