MAT1732B Devoir 1 -7211288

Question 1.

$\int \sin^{-1} x\,dx$ u= $\sin^{-1} x$ v' = 1

u' = $\frac{1}{\sqrt{1-x^2}}$ v = x

$= x\sin^{-1} x - \int \frac{x\,dx}{\sqrt{1-x^2}}$ w =1- x^2 , dw/dx = -2x , dx = dw/-2x

$= x\sin^{-1} x + \frac{1}{2}\int \frac{dw}{\sqrt{w}}$

$= x\sin^{-1} x + \frac{1}{2}\int w^{(-1/2)}\,dw$

$= x\sin^{-1} x + \sqrt{w} + c$

$= x\sin^{-1} x + \sqrt{1-x^2} + c$

Question 2.

$\int_0^1 x^5 e^{(-x^3)}\,dx$

$= \int \frac{x^5}{e^{(x^3)}}\,dx$ u = x^3 , du/dx = $3x^2$, dx = $\frac{du}{3x^2}$

$= \int \frac{x^5\,du}{e^u 3x^2}$

$= \frac{1}{3}\int \frac{x^3}{e^u}\,du$

$= \frac{1}{3}\int \frac{u}{e^u}\,du$ w = u et w' = du , v' = $e^{(-u)}$ et v = $-e^{(-u)}$

$= \frac{1}{3}[-ue^{(-u)} + \int e^{(-u)}\,du]$

$= \frac{1}{3}[-ue^{(-u)} - e^{(-u)}] + c$

$= \frac{-1\,e^{(-u)}}{3}[u+1] + c$

$= \frac{-e^{(-x^3)}}{3}[x^3+1] + c$ =F(x)

F(1) = $\frac{-e^{(-1^3)}}{3}[1^3+1]$

= -0.245

F(2) = $\frac{-e^{(-0^3)}}{3}[0^3+1]$

= -1/3

F(1)-F(0)

= -0.245 - (-1/3)

= 0.088

Question 3.

a) Pas = Δx = $\frac{2-1}{4}$ = 1/4

$x_0=1$ $x_1=x_0+\Delta x$ $x_2=x_1+\Delta x$ $x_3=x_2+\Delta x$ $x_4=x+3+\Delta x$

= 1 + 1/4 = 5/4 + 1/4 = 6/4 + 1/4 = 7/4 + 1/4

= 5/4 = 6/4 =7/4 = 8/4

$[f(x_1)+f(x_2)+f(x_3)+f(x_4)]\Delta x$

= $[\ln(5/4)+\ln(6/4)+\ln(7/4)+\ln(8/4)]\frac{1}{4}$

= 0.47

b) Pas = Δx = $\frac{2-1}{10}$ = 1/10

$x_0=1$ $x_1=x_0+\Delta x$ $x_2=x_1+\Delta x$ $x_3=x_2+\Delta x$ $x_4=x_3+\Delta$ $x_5=x_4+\Delta$ $x_6=x_5+\Delta x$

= 1+ 1/10 =11/10+1/10 =12/10+1/10 =13/10+1/10 =14/10+1/10 =15/10+1/10

= 11/10 =12/10 = 13/10 = 14/10 = 15/10 = 16/10

$x_7=x_6+\Delta x$ $x_8=x_7+\Delta x$ $x_9=x_8+\Delta x$ $x_{10}=x_9+\Delta x$

= 16/10+1/10 = 17/10+1/10 = 18/10+1/10 = 19/10+1/10

= 17/10 = 18/10 = 19/10 = 2

$[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)+f(x_7)+f(x_8)+f(x_9)+f(x_{10})]\Delta x$

= $[\ln(\frac{11}{10})+\ln(\frac{12}{10})+\ln(\frac{13}{10})+\ln(\frac{14}{10})+\ln(\frac{15}{10})+\ln(\frac{16}{10})+\ln(\frac{17}{10})+\ln(\frac{18}{10})+\ln(\frac{19}{10})+\ln(2)]\cdot(\frac{1}{10})$

= 0.421

c) $\int_1^2 lnx\,dx$

= $\int lnx\,dx$ u = lnx et u' = 1/x , v' = 1 et v = x

= $xlnx-\int(\frac{x}{x}dx)$

F(x)= $xlnx-x+c$

F(2) = 2ln2 -2

F(1) = ln2 -1

F(2) – F(1)

= 2ln2 -2 – [ln1-1]

= 0.39

Question 4.

a) $f(t) = 100te^{(-t/10)}$

$f'(t) = 100te^{(-t/10)}(\frac{-1}{10}) + 100e^{(-t/10)}$

$0 = 10e^{(-t/10)}[-t+10]$

$-t + 10 = 0$ $\qquad$ $10e^{(-t/10)} = 0$

$t = 10$ jours $\qquad$ impossible

b) $f(t) = 100te^{(-t/10)}$

$f(10) = 100(10)e^{(-10/10)}$

$= 367.9$ cas

c) $F(t) = \int 100te^{(-t/10)}$

$u = 100t$ et $u' = 100$, $v' = e^{(-t/10)}$ et $v = -10e^{(-t/10)}$

$w = -t/10$

$dw/dt = -1/10$

$dx = -10dw$ donc, $v = \int -10e^{w}\,dw$

$v = -10\int e^{w}\,dw$

$v = -10e^{(-t/10)} + c$

$F(t) = -1000te^{(-t/10)} - \int -1000e^{(-t/10)}\,dt$

$= -1000te^{(-t/10)} + 1000\int e^{(-t/10)}\,dt$

$= -1000te^{(-t/10)} + 1000\cdot(-10e^{(-t/10)}) + c$

$= -1000te^{(-t/10)} - 10000e^{(-t/10)} + c$

d) $\lim_{(t\to\infty)} F(t)$

$= \lim_{(t\to\infty)}[-1000te^{(-t/10)} - 10000e^{(-t/10)}]$

$= -1000(10000)e^{(-10000/10)} - 10000e^{(-10000/10)}$

$= 0$