York University

Department of Mathematics and Statistics

Math 1505: Mathematics for Life and Social Sciences

Final Examination April 15, 2012

Name: _____
Last Given

Student Number: _____

- **Answer all questions**
- **All work must be shown**
- **Total points 200**
- **Nongraphing, nonprogrammable calculator permissible**

Instructors:
Bugajska: Sec A, MWF @ 10:30 (SLH F)
Purzitsky: Sec B, MWF @ 9:30 (Tel 0016)
Bugajska: Sec D, MWF @ 7pm (SLH A)
Pietrowski: Sec E, Tues @ 10:30 (CLH A)
Purzitsky: Sec G, MWF @ 10:30 (Tel 0016)
Pietrowski: Sec H, MWF @ 9:30 (CLH C)

Print in the box, in bold letter, your section

Question	Pts	Score
1	4	
2	4	
3	6	
4	24	
5	24	
6	6	
7	6	
8	6	
9	20	
10	6	
11	7	
12	35	
13	8	
14	12	
15	16	
16	16	
Total	200	

1[4 points] Find the largest domain for the function

$$f(x)=\ln\left(\frac{2-x}{x+1}\right).$$

2[4 points] Solve $2\sin\theta\tan\theta+\tan\theta=0$ on $[0,2\pi)$.

3[6 points] After 150 days, a particular radioactive substance decays to 30% of its original amount. In how many days (to two decimal places) will the substance decay to 60% of its original amount?

4[24 points] Determine the following limits:

a) $\lim_{x\to-\infty} \frac{3e^{x}+e^{-x}}{4e^{x}-7e^{-x}}$

b) $\lim_{x\to\frac{3}{2}} \frac{2x^{2}+3x-9}{2x^{2}+x-6}$

c) $\lim_{x\to\infty} \dfrac{\frac{2}{x^2}-\frac{3}{x}}{\frac{5}{x^2}+\frac{7}{x}}$

d) $\lim_{x\to\infty}\left(\sqrt{2x^2+x}-\sqrt{2x^2+1}\right)$

5[24 points] Find $\frac{dy}{dx}$. **Do not simplify.**

(a) $\sqrt{3x}+5x+\frac{1}{x^3}$.

(b) $\left(x^2+\frac{1}{x}\right)^5$

(c) $\frac{x^3-2x}{x^4-5x}$

(d) $\left(x^2+1\right)^{\sqrt{x}}$

(e) $\cos x\sqrt{1+\sin x}$

(f) $y=\int_1^{x^3}\frac{1}{e^u+1}du$

6[6 points] Find an equation of the tangent line to the curve $x^3-\frac{2x}{y}+y^3=11$ at (2, -1).

7[6 points] Let $f(x)=\sqrt{5+4x^2}, x\geq 0$. Find $\frac{d}{dx}\left(f^{-1}(x)\right)$ at x=3.

8[6 points] If the measurement of the diameter of a circle is accurate to within 5%, how accurate will the calculation of the area of the circle be? Recall: area of the circle is $\pi\left(\frac{d}{2}\right)^2$ where d is the diameter.

9[20 points] Let $f(x)=x^4-2x^2$. The first and second derivates are $f'(x)=4x^3-4x$, $f''(x)=12x^2-4$.

a)Find the intervals where the function is increasing and where the function is decreasing. Find the coordinates of the local extrema;

$f(x)=x^4-2x^2$, $f'(x)=4x^3-4x$, $f''(x)=12x^2-4$

b)Find the intervals where the graph of the function is concave up and where the graph of the function is concave down. Does the function have inflection points. If yes, find the coordinates of the inflection point(s).

c) Sketch the graph of the function.

10[6 points] A rectangular study plot is to be enclosed by 625 meters of fencing. What are the dimensions of the study plot that will have the largest area? Justify your answer.

11[10 points] Calculate the area of the region bounded by $y=\sqrt{x+2}$, $y=x$ and $y=0$.

12[35 points] Evaluate the following integrals.

(a) $\int x^2\sqrt{x+1}dx$

(b) $\int_1^4 \frac{1}{\sqrt{x}}e^{\sqrt{x}}dx$

(c) $\int x\cos dx$

$\int x\cos x\,dx$

(d) $\int \frac{x^2+1}{x^3+2x} dx$

(e) $\int_1^{\infty} \frac{1}{e^x} dx$

13[8 points] Solve the following system of linear equations:

$$\begin{cases} -2x+4y-z=8 \\ x+7y+2z=5 \\ 3x+3y+3z=-3 \end{cases}$$

14 [12 points] Suppose A and B are events with P(A)=.5, P(B)=.6 and $P(A \cup B)=.8$, Find

(i) $P(B^c)$

(ii) P(A|B)

(iii) P(B|A)

(iv) Are A and B independent? Explain.

15 [16 points]

(a) How many ways can a committee of three people be formed from a group of eight people if two of the people do not want to serve together?

(b) You roll a fair die twice. Find the probability that the minimum of the two numbers is four or more.

(c) An urn has 5 white balls and 3 green balls. Three balls are chosen at random without replacement. What is the probability that at least two out of the three balls are green?

(d) A coin is tossed four times. Find the probability of no more than three heads given that at least one toss resulted in a head.

16[13 points] A screening test for a disease shows a positive test result in 95% of all cases when the disease is actually present and in 10% of all cases when it is not. Assume that the prevalence of the disease in the population is 2 in 50.

(a)If such a test is administered to a randomly chosen individual, what is the probability that the test result is positive?

(b)If an individual tests negative, what is the probability that the individual actually has the disease?