CONCORDIA UNIVERSITY

Department of Mathematics and Statistics

Course	Number	Section
Math	204	EC
Examination	**Date**	**Pages**
Alternate Final	Winter 2010	3
Instructors		**EC Course Examiners**
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Instructions

- Answer all ten questions.
- Only approved calculators are allowed.
- No other material is allowed.

Evaluation

All questions are of equal value. The examination counts for 50% towards your final grade.

Questions

Question 1

(a) Let $\mathbf{u} = (1, 1, 1)$. Find vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbb{R}^3$ that are orthogonal to $\mathbf{u}$ and to each other.

(b) Find the area of the parallelogram with vertices $(0, 0, 0)$, $(1, 2, -2)$, $(2, 3, 0)$ and $(3, 5, -2)$.

Question 2

Let $\mathbf{x} = (-2, 4, 1)$ and $\mathbf{y} = (1, 2, -1)$.

(a) Find scalars r and s such that $r\mathbf{x} + s\mathbf{y} = (11, -2, -8)$.

(b) Find a vector $\mathbf{z}$ such that $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is a basis for $\mathbb{R}^3$. Justify your answer.

Question 3

(a) Let $\mathbf{x} = (2, 1, -2)$ and $\mathbf{y} = (3, -3, -3)$. Find the orthogonal projection $\mathbf{z}$ of $\mathbf{x}$ on $\mathbf{y}$ and a vector $\mathbf{w}$ orthogonal to $\mathbf{y}$ such that $\mathbf{x} = \mathbf{z} + \mathbf{w}$.

(b) Find the distance from the point $(4, -3)$ to the line $3x + 5y - 2 = 0$.

Question 4

Let

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 5 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \end{bmatrix}$$

Find a basis for the solution space of the homogeneous system of linear equations $AX = \mathbf{0}$.

Question 5

(a) Rewrite the system

$$\begin{cases} x + y - 3z = 7 \\ x + z = 0 \\ 3x + y = 2 \end{cases}$$

as a matrix equation of the form $Ax = b$, and show that

$$x = \begin{bmatrix} 1 & 3 & -1 \\ -3 & -9 & 4 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 0 \\ 2 \end{bmatrix}$$

is a solution of $Ax = b$.

Question 6

(a) Find the equation of the plane that contains the point $(2, -2, 1)$ and is perpendicular to the line with parametric equations $x = t - 1$, $y = 3t + 2$, $z = t + 1$.

(b) Find parametric equations for the line in $\mathbb{R}^3$ that contains the points $(1, 1, -1)$ and $(2, 1, 2)$.

Question 7

Convert the linear system

$$L = \begin{cases} 2x + 6y - z = 4 \\ 3x + 9y = 15 \\ -3x - 9y + z = -9 \end{cases}$$

to an augmented matrix in the variables x, y, z and use elementary row operations to find the solutions of L.

Question 8

Find two 2×2 matrices A and B for which the vector

$$BA \begin{bmatrix} x \\ y \end{bmatrix}$$

is the reflection of the vector

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

about the line $y = x$, followed by a counterclockwise rotation of the vector by $\pi/3$ radians and calculate the vector

$$BA \begin{bmatrix} x \\ y \end{bmatrix}$$

(Hint: $\cos(\pi/3) = \dfrac{1}{2}$ and $\sin(\pi/3) = \dfrac{1}{2}\sqrt{3}$.)

Question 9

Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & -6 \\ 3 & 0 & -2 \end{bmatrix}$$

Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

Question 10

Let

$$M = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 0 & 1 \end{bmatrix}$$

(a) Verify that the matrix

$$N = \begin{bmatrix} 3 & 2 & -2 \\ -4 & -1 & 2 \\ -1 & -1 & 1 \end{bmatrix}$$

is the inverse of M by showing that the two matrix products MN and NM are both the 3×3 identity matrix.

(b) Find the matrix C such that $MC = B$.