Econ 302, Winter 2013
Assignment 1: Pure Competition
Suggested Answers

Part 1: True/False questions. Fully explain your answer.

1. FALSE. The minimum AVC can be found as

$$\min_y AVC = 10 - 2y + 0.5y^2$$

with the FOC being

$$\frac{\partial AVC}{\partial y} = 0 \Rightarrow -2 + y = 0 \Rightarrow y = 2.$$

Therefore,

$$AVC \min = 10 - 2(2) + 0.5(2)^2 = \$8.$$

Since $p = \$6 < \$8$ the firm should shut-down.

2. FALSE. The vertical distance between the average total cost and the average variable cost both evaluated at y^* expresses the average fixed cost.

3. FALSE. Because the lonr-run supply of a competitive industry is horizontal at a price equal to $AVC \min$ (zero profit condition), the firms cannot afford to pay anything more. The entire tax will be paid by the consumers (the firms will fully pass the tax on them).

4. TRUE. In the long-run the firm stays in the market as lng as it realizes normal profits, i.e., $\Pi \geq 0$. Given the formula for the producer surplus we get

$$PS \geq 0 \Rightarrow \Pi + F \geq 0$$

But in the long-run there is no fixed cost. So, the above expression yields

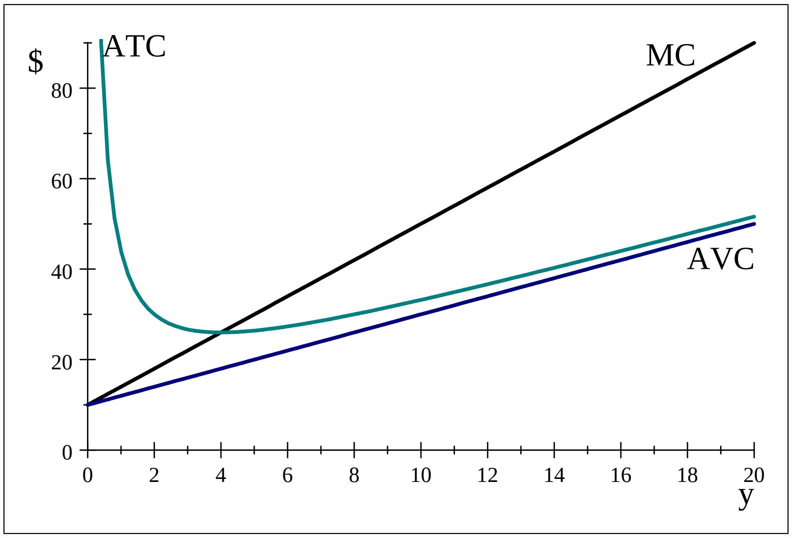
$$PS \geq 0 \Rightarrow \Pi \geq 0$$

Part 2: Numerical problems

1. (a) The fixed cost is $F = 32$ and the variable cost is $c_v[y_i] = 10y_i + y_i^2$. Therefore, we get

- average fixed cost: $AFC[y_i] = F/y_i \Rightarrow AFC[y_i] = 32/y_i$
- average variable cost: $AVC[y_i] = c_v[y_i]/y_i \Rightarrow AVC[y_i] = 10 + 2y_i$
- average total cost: $ATC[y_i] = c[y_i]/y_i \Rightarrow ATC[y_i] = 32/y_i + 10 + 2y_i$
- marginal cost: $MC[y_i] = \partial c[y_i]/\partial y_i \Rightarrow MC[y_i] = 10 + 4y_i$

Graphically, we get



(b) Since the marginal cost is always greater than the average variable cost (by looking at the graph we can confirm that) the entire marginal cost is the inverse supply of the farmer. That is, the farmer will be producing according to

$$MC\left[y_i\right] = p \Rightarrow 10 + 4y_i = p \Rightarrow y_i = 0.25p - 2.5 \Rightarrow S_i\left[p\right] = 0.25p - 2.5$$

The total supply is

$$S\left[p\right] = 1,000S_i\left[p\right] \Rightarrow S\left[p\right] = 1,000\left(0.25p - 2.5\right) \Rightarrow S\left[p\right] = 250p - 2,500.$$

(c) The equilibrium price is the one that equates quantity demanded to quantity supplied, i.e.,

$$S\left[p\right] = D\left[p\right] \Rightarrow 250p - 2,500 = 9,500 - 50p \Rightarrow 300p = 12,000 \Rightarrow p^* = \$40.$$

Each farmer will be selling (according to the individual supplies found in part b)

$$S_i\left[p\right] = 0.25p - 2.5 \Rightarrow y_i = 0.25\left(40\right) - 2.5 \Rightarrow y_i^* = 7.5$$

Finally, the profits are

$$\Pi_i = py_i - \left(32 + 10y_i + 2y_i^2\right) \Rightarrow \Pi_i = 40\left(7.5\right) - \left(32 + 10\left(7.5\right) + 2\left(7.5\right)^2\right) \Rightarrow \Pi_i^* = 80.5$$

(d) Since the firms are making positive profits in the short-run, we are expecting more firms to enter in the long-run! The long-run price is the break-even price, i.e., the minimum value that the ATC can take. To find the ATC min we set

$$\begin{aligned} ATC &= MC \Rightarrow 32/y_i + 10 + 2y_i = 10 + 4y_i \Rightarrow \\ 32/y_i + 2y_i &= 4y_i \Rightarrow 32/y_i = 2y_i \Rightarrow ...y_i = 4 \end{aligned}$$

Therefore, when $y_i = 4$ the ATC will be

$$ATC = 32/4 + 10 + 2\,(4) \Rightarrow ATC\,\text{min} = \$26.$$

Given that the market price in the long-run is $p_{LR} = \$26$, we can identify:

- how much a single firm will be producing in the long-run, by plugging this price in the firm's supply we got in part (b). That is,

$$S_i\,[p] = 0.25\,(26) - 2.5 \Rightarrow y^*_{LR,i} = 4$$

- how much the quantity demanded will be. That is,

$$D\,[p] = 9,500 - 50\,(26) \Rightarrow Y = 8,200$$

Therefore, since a single firm produces 4 units while the market wants 8200, the number of firms in the industry will have to be

$$n = \frac{8200}{4} \Rightarrow n = 2,050.$$

2. (a) We get

- variable cost: $AVC\,[y] = c_v\,[y]\,/y \Rightarrow 10 + 4y = c_v\,[y]\,/y \Rightarrow (10 + 4y)\,y = c_v\,(y_i) \Rightarrow c_v\,[y] = 10y + 4y^2$
- marginal cost: $MC\,[y_i] = \partial c_v\,[y]\,/\partial y \Rightarrow MC\,[y] = 10 + 8y$

However, to get the rest of the costs we need to find the first cost. To do so, note that according to this example the firm is making $\Pi = 14$ when the price is $p = 42$. Given the MC of the firm we can get the firm's supply function, i.e.,

$$MC = p \Rightarrow 10 + 8y = p \Rightarrow y = 0.125p - 1.25$$

and given the price we get

$$y = 0.125p - 1.25 \Rightarrow y = 0.125\,(42) - 1.25 \Rightarrow y = 4$$

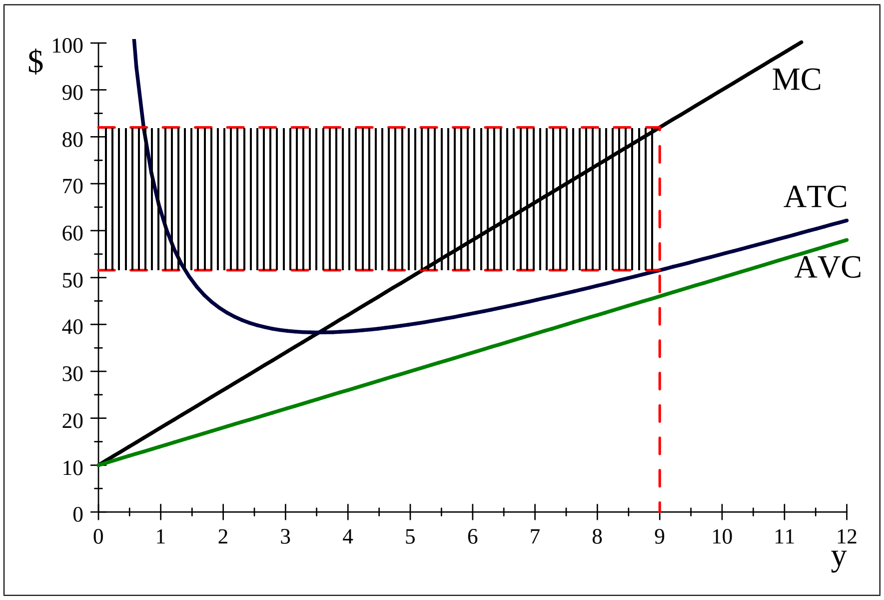
Finally,

$$\begin{aligned} \Pi &= py - c\,[y] \Rightarrow \Pi = py - c_v\,[y] - F \Rightarrow \Pi = py - \left(10y + 4y^2\right) - F \Rightarrow \\ 14 &= 42\,(4) - \left(10\,(4) + 4\,(4)^2\right) - F \Rightarrow F = \$50 \end{aligned}$$

Now we can get

- total cost: $c\,[y] = c_v\,[y] + F \Rightarrow c\,[y] = 10y + 4y^2 + 50$
- average total cost: $ATC\,[y] = c\,[y]\,/y \Rightarrow ATC\,[y] = 10 + 4y + 50/y$

Graphically, we get



(b) Let the market price be \$82. How much will the firm produce? Determine the associated profit and producer surplus. Show your results in the graph you constructed for part (a). Given the supply found in part (a), when $p = \$82$ we get

$$y = 0.125p - 1.25 \Rightarrow y = 0.125\,(82) - 1.25 \Rightarrow y - 9.$$

Therefore, we get

$$\Pi = 82\,(9) - \left(10\,(9) + 4\,(9)^2 + 50\right) = \$274,$$

and

$$PS = \Pi + F \Rightarrow \Pi = 274 + 50 = \$324$$

This profit is depicted in the graph above as the shaded area between the price and the average total cost at $y = 9$ (if you plug $y = 9$ in the ATC you will get that $ATC\,(9) = 51.556$). The producer surplus is just the triangle area above the MC and under the price.

3. For type 1 firms we get

$$MC\,[y_1] = \partial c\,[y_1]\,/\partial y_1 \Rightarrow MC\,[y_1] = 5 + y_1$$

Since this MC is always greater than the AVC it will be the entire MC identifying the inverse supply of a type 1 firm. That is,

$$MC\,[y_1] = p \Rightarrow 5 + y_1 = p \Rightarrow y_1 = p - 5$$

Since there are 150 firms of type 1, the total supply of all type 1 firms will be

$$S_{1,Total} = 150\,(p-5) \Rightarrow Y_1 = 150p - 750$$

For type 2 firms we get

$$MC\,[y_2] = \partial c\,[y_2]\,/\partial y_2 \Rightarrow MC\,[y_2] = 1 + 1.5y_2$$

Since this MC is always greater than the AVC it will be the entire MC identifying the inverse supply of a type 2 firm. That is,

$$MC\,[y_2] = p \Rightarrow 1 + 1.5y_2 = p \Rightarrow y_2 = (2/3)p - (2/3)$$

Since there 350 firms of type 2, the total supply of all type 2 firms will be

$$S_{2,Total} = 360\,((2/3)\,p - (2/3)) \Rightarrow Y_2 = 240p - 240$$

Given these individual supplies, the market supply when both firms are in the market (i.e., $p \geq \$5$) is

$$S\,(p) = S_{1,Total}\,[p] + S_{2,Total}\,[p] \Rightarrow S\,[p] = (150p - 750) + (240p - 240) \Rightarrow S\,[p] = 390p - 990$$

However, one can verify that for any price $p \in (1,5)$ only firms of the second type are in the market. That is over that range the market supply is the supply of firm 2, i.e.,

$$S\,[p] = S_{2,Total}\,[p] = 240p - 240$$

In summary

$$S\,[p] = \begin{cases} 390p - 990, & \text{if } p \geq 5 \\ 240p - 240, & \text{if } 1 \leq p \leq 5 \\ 0, & \text{otherwise} \end{cases}$$