CONCORDIA UNIVERSITY
FACULTY OF ENGINEERING AND COMPUTER SCIENCE
DEPARTMENT OF MECHANICAL AND INDUSTRIAL ENGINEERING

FINAL EXAMINATION - FALL 2007
MECHANICAL ENGINEERIG DESIGN - MECH 441

Duration of the exam: 3 hours; Materials allowed: textbook only, non-programmable calculators (one of the two approved types), return the question paper within the answer book.

Your family name: . . . .

Your name: . . . .

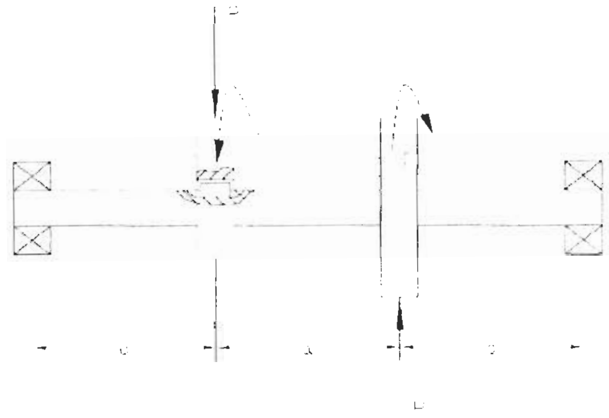
Your Concordia I.D.#: . . . .

Very important information for the examination period – please read carefully.

Please read carefully this paragraph before proceeding with the solutions. Please, make sure that you respect the requirements of this examination. You have to solve all five (5) problems to attempt the full grade. **Each solution must start on a new page in the answer book marked in the upper left corner by the number representing the question for which the answer is attempted.** It ***does not matter*** the order in which you solve the problems. After solving the problem, make sure that you mark the response to each question in the problem in the summary sheet. I assume that no writing means no attempt and the grade for the respective problem will be assigned accordingly. Use three decimal points accuracy in calculations. Be aware that the only allows materials are the ones listed above. Make sure that you do not have any other materials under your possession. Also make sure that you do not have in the book excessive amount of extra information. Such situation may be regarded a cheating. The book should be used only for reference during the exam. NO CELLULAR PHONES ARE ALLOWED IN YOUR POSSESSION DURING THE EXAM. *Please make sure that you respect all the above fundamental requirements.*
Caveat: please, pay attention to calculations and units.
The questions have given all necessary data such that you do not have to assume anything. However, it may be the case that you have doubts about the information that you has at the hand. For these circumstances, here are the clarifications. All described situations are happening on the ground, at the normal pressure and temperature unless other way specifically indicated. In case a pressure value is provided, that pressure represents the gauge pressure. When an optimum design is targeted, a specific value representing a unique number can be identified using simple judgement and the information that is available in the textbook. *Do not assume when you can calculate.*
If a specific value is indicated through a value or a clue, do not assume another number (for an example, the safety factor is given as value N_d; please, do not select another value for the safety factor under the assumption that the new selected value is better than the one that has been indicated).

Question #1 (30 marks)

The below figure shows a shaft which is under loading induced by the transmission elements.



The shaft is subjected to a loading P and a constant torque T between the two rollers. For the provided data, a) find the safety factor in fatigue using ASME standard assumptions with regards to stress concentration factor K_{fsm}. b) The two rollers are each 4 in diameter and weight 5 lbs. Find the critical speed for torsional vibration of the rotary system. c) In case you wish to increase the critical speed, what would you do as a designer, when you are constrained to make no changes in the geometry of the system between the two disks (disks and shaft dimensions cannot be changed as well as the distance between disks). Reason your statement using analytical formulation.

Given: material - AISI 1030 quenched and tempered 1000° F, diameter of the shaft – d = 1in, distance a = 2 in, concentrated load - P = 600 lbf, torque T = 1100 lbf×in, the stress concentration factor in the key - $K_f = 2.5$, the shaft is machined, it operates at the room temperature, and it is required a reliability of 90%, the mass of a disk - m_d = 5 lb, diameter of the disk – 4 in. Ignore the shear effect.

Question #2 (20 marks)

A spring must be designed from specific clues that are provided by the results of the other machine elements interacting with the spring. The stiffness of the spring must be k = 1619 N/m and also it is known that the operation frequency of the mechanism is 73 Hz (the resonant frequency of the spring must be 13 times higher). A spring factor C = 8 has been selected. a) Determine the diameter of the wire d and the number of the active coils N_a for the spring. b) The geometric constraint imposes you a preload corresponding to an initial deflection of y_1 = 1.2 cm. What is the maximum stress in the spring according to Wahl's formulation under the force due to the initial deflection y_1? c) The maximum load on the spring is due to a stroke of y_2 = 3.7 cm. What is the static safety factor N_s for the spring when the stress in the coil is calculated under Wahl formulation under the load induced by the maximum stroke y_2? For the spring, A227 wire is used.

Given data: G = 80.5 GPa

Question #3 (20 marks)

Two flanges each ½" thick must be held together and a total force of 9,000 lbf must be applied through fastening to avoid the separation of the parts. Three UNC bolts must be used. Assume that the load is distributed equally among the bolts and that the available bolts are SAE Grade 5. a) Indicate the size type of the bolts (ANSI type) such that the fastening is performed at a tightening force that corresponds with a stress in the bolt of 75% of the proof strength. b) Determine the stiffness C of the joint, knowing that the flanges are made form grey cast iron class 40. c) Find the separation factor N_{sep} of the joint for an applied pull force between flanges of 3600 lbf. Assume in calculations that the bolt is threaded till under the head in the joint. **Hint:** the stiffness of the joint should be evaluated using Wileman empirical formulation.

Question #4 (15 marks)

A square key has width a fraction "s" of the shaft diameter "d". The shaft and the key are made of materials that are equally strong (same S_y) with a yield point value in shear equal to ½ the yield point is tension.

a) Find the requested length of the key in terms of the symbolical provided values.
b) How is the length changing with the strength of the key material if $S_{y\_key} = \zeta \times S_{y\_shaft}$? ($\zeta < 1$)

Question #5 (15 marks)

A shaft is subjected to both fluctuating torque and bending moment. You are the designer in charge with the machine elements and you realize that the fatigue safety factor for the shaft in a section of interest is close but not as large as the specified value. The way that you performed the design enables you to transfer the loads such that transfers between σ_m and σ_a can be carried in the shaft. However, the transfer relation between the two stresses is given by the following relationships: $\sigma_a = k_a \times S_e$, $\sigma_m = k_m \times S_{ut}$, ($k_a + k_m = k$ = constant). You also recall that in the augmented modified Goodman line, the portion carrying large σ_m and small σ_a is shortened by the yield line, which may be exactly your case of below performance design result. You need to establish what is the ratio of σ_a/σ_m such that you secure your design in a slightly higher safety factor region. For calculation purposes assume the following data: $S_y = 0.8 \times S_{ut}$, $S'_e = 0.4 \times S_{ut}$. The design requires usage of the augmented modified-Goodman diagram. Use the graph for solution along with the mathematical reasoning in finding the ratio σ_a/σ_m.

Solution pb #1.

Material AISI 1030 Q&T @ 1000F, $S_{ut} = 97$ Kpsi, $S_y = 75$ Kpsi / p. 948

$P = 600$ lbf $\quad T = 1100$ lbf in

$d = 1$ in $\quad a = 2$ in $\quad , K_f = 2.5$

$S_e' = 0.5\, S_{ut} = 48.5$ Kpsi

a)

$$N_f = \frac{\pi d^3}{32\sqrt{\left(K_f \frac{M_a}{S_e'}\right)^2 + \frac{3}{4}\left(\frac{T_m}{S_y}\right)^2}} \quad [9.66]$$

$$S_e = C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S_e'$$

$C_{load} = 1$

$C_{size} = 0.869\, d^{-0.097} = 0.869$

$C_{surf} = A\,(S_{ut})^b \quad A = 2.7,\ b = -0.265 \qquad C_{surf} = 0.803$, $S_{ut} = 97$

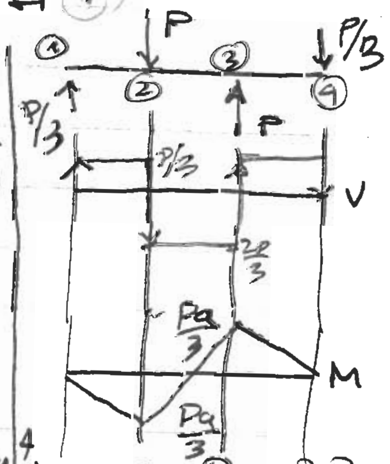
$C_{temp} = 1$

$C_{reliab} = 0.897 \qquad \Pi\, C_i = 0.869 \times 0.803 \times 0.897 = 0.626$

$S_e = 0.626 \times S_e' = 30.36$ kpsi

$$N_f = \frac{\pi \cdot 1^3}{32\sqrt{\left(2.5 \cdot \frac{400}{30{,}360}\right)^2 + \frac{3}{4}\left(\frac{1100}{75000}\right)^2}}$$

$$\boxed{N_f = 2.781}$$



$M_1 = 0 = Pa - 2Pa + 3R_2 a$

$R_2 = \frac{P}{3}$

$M_4 = 0 = 3R_1 a - 2Pa + Pa$

$R_1 = \frac{P}{3}$

$M_{max} = \frac{Pa}{3} = \frac{600 \cdot 2}{3} = 400$ lb in

b) $m = 5 \quad r = 2$

$I_m = \frac{m r^2}{2} = 10$ lb in² $\quad J = \frac{\pi d^4}{32} = 0.098$ in⁴

$K_s = \frac{G \cdot J}{a} = \frac{1.154 \times 10^7 \cdot 0.098}{2} = 5.664 \times 10^5$ lb/in

$f = \sqrt{\frac{2 \cdot K_s}{I_m}} = 336.569$ rad/s $= 53.567$ Hz

c) Since the geometry cannot be changed,

K should be increased → not really possible 1

$$k_s = \frac{G \cdot J}{a} \quad 1$$

or I_m reduced ⇒ diam. remains same but the mass could be reduced ⇒ 1

Reduce the mass of the disks. !

$$I_m = \frac{mr^2}{2} \quad 2$$

r = const 1, m ↓ ⇒ freq ↑ 1

Solution Pb # 2

$G = 80.5\ GPa$ $\qquad C = 8$

a)

$$\begin{cases} k = \dfrac{d^4 G}{8D^3 \cdot N_a} \\[2ex] f_m = \dfrac{2}{\pi \cdot N_a} \cdot \dfrac{d}{D^2}\sqrt{\dfrac{G \cdot g}{32\gamma}} \\[2ex] D = c \cdot d \end{cases}$$

(1.5)

$$K = \frac{d^{\cancel{4}} G}{8C^3 \cancel{d^3} N_a} \qquad d = \frac{8KC^3 N_a}{G}$$ (2)

$$f_m = \frac{2}{\pi \cdot N_a} \cdot \frac{\cancel{d}}{c^2\left(\frac{8KC^3 N_a}{G}\right)^{\cancel{2}}}\sqrt{\frac{G}{32\rho}}$$

$$f_m = \frac{G}{4\pi N_a^2 K c^5}\sqrt{\frac{G}{32\rho}}$$

$\boxed{N_a = 8.5}$ (2)

$$d = \frac{8KC^3 N_a}{G}$$

$\boxed{d = 7.002\times10^{-4} = 0.7\ mm}$ (2)

b) $$y_1 = \frac{8F_1 D^3 N_a}{d^4 G} = 0.012 \Rightarrow F_1 = \frac{d^4 G y_1}{8D^3 N_a} = 20)$$ (2)

wahl #: $$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.184$$ (2)

$$\tau_{F_1} = K_w \cdot \frac{8FD}{\pi d^3} = \underline{9.845\times10^8\ Pa}$$ (2)

c) $$y_2 = \frac{8F_2 D^3 N_a}{d^4 G} = 0.037 \Rightarrow F_2 = \frac{d^4 G y_2}{8D^3 N_a} = 60\ N$$

$$\tau_{max_{F_2=60N}} = \frac{8F_2 D}{\pi d^3} \cdot k_w$$

$$\tau_{max_2} = 2.958 \times 10^9 \; Pa$$

2

safety. wire strength:

$$S_{ut} = A d^b \qquad A = 1753.3 \times 10^6 \; (A227)$$
$$b = -0.1822$$

$$S_{ut} = 6.587 \times 10^9 \; Pa$$

0.5

3

$$N_S = \frac{S_{ut}}{\tau_{max_2}} = \frac{6.587 \times 10^9}{2.954 \times 10^9} = \underline{\underline{2.23}}$$

Solution Pb # 3

a) $F = 9000$ lbf $\quad E = 30 \times 10^6$ psi $\quad S_y = 92 \times 10^3$ psi

$n = 3 \quad S_{ut} = 120 \times 10^3$ psi $\quad S_p = 85 \times 10^3$

$$\sigma_a = 0.75 \cdot S_p = \frac{F_0}{A_b} \Rightarrow A_b = \frac{F_0}{0.75 S_p}$$

$$A_b = 0.047 \text{ in}^2$$

This size corresponds the size 5/16" × 3 bolts

$$d = \frac{5}{16}$$

5

b) grey cast $E_c = 14.5 \times 10^6$ (cast iron) $\quad A_{bm} = 0.0524$ (area of the bolt standard size)

$\cancel{A_b =}$ p. 849, table 14-9 → rel. 14.17b.

$A = 0.77871$

$b = 0.61616$

$l_m = 1$

$$k_m = d \cdot E_c \cdot A e^{\frac{b \cdot d}{l_m}}$$

$$k_m = 4.278 \times 10^6 \ \frac{lb}{in}$$

$$k_b = \frac{A_{bm} \cdot E}{l_m} = \quad k_b = 1.572 \times 10^6 \ \frac{lb}{in}$$

10

$$C = \frac{k_b}{k_m + k_b} = 0.269$$

c) $P_{sp} = \frac{3600}{3} = 1200$ lb $\quad F_b = C \cdot P_{sp} = 322.47$ lb

$F_m = (1-C) P_{sp} = 877.53$ lb

$$N_f = \frac{F}{P_{sp}(1-C)} = \boxed{3.419}$$

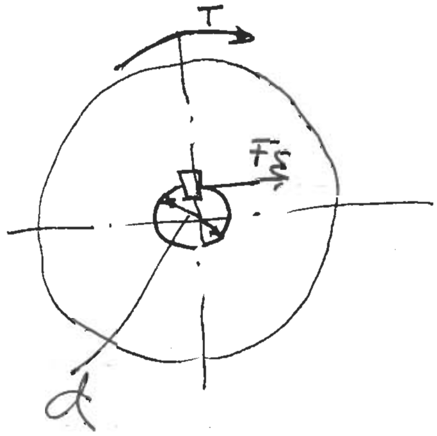
5

Solution P6 #4 – 15 marks

$w = s \cdot d$

$S_{y_s} = 0.5\, S_{y_t}$

$F_S = \dfrac{T}{\frac{d}{2}} = \dfrac{2T}{d}$ → the maximum torque at failure



a) The shear stress in key:

$$\tau_s = \frac{F_S}{w \cdot \underline{\underline{l}}} = \frac{F_S}{s \cdot d \cdot \underline{\underline{l}}} = \frac{S_{ut}}{2}$$

The torsion stress in the shaft:

$$\tau_t = \frac{T}{I_p} = \frac{T}{\frac{\pi d^4}{32}} = \frac{32T}{\pi d^4} = \frac{S_{ut}}{2}$$

The key must fail before the shaft so at the limit

$$\tau_s = \tau_t$$

$$\frac{2\cancel{T}}{S d^2 \underline{\underline{l}}} = \frac{32\cancel{T}}{\pi d^{\cancel{4}\,2}} \qquad l = \frac{2\pi d^2}{32 S} = \frac{\pi d^2}{16 S}$$

3 $\boxed{l \le \dfrac{\pi d^2}{16 S}}$ to make sure that the key will fail first (befor the shaft fails)

6)

$$\tau_s = \frac{F_s}{w \cdot l} = \frac{F_s}{s \cdot d \cdot l} = \xi \frac{S_{o}t}{2}$$

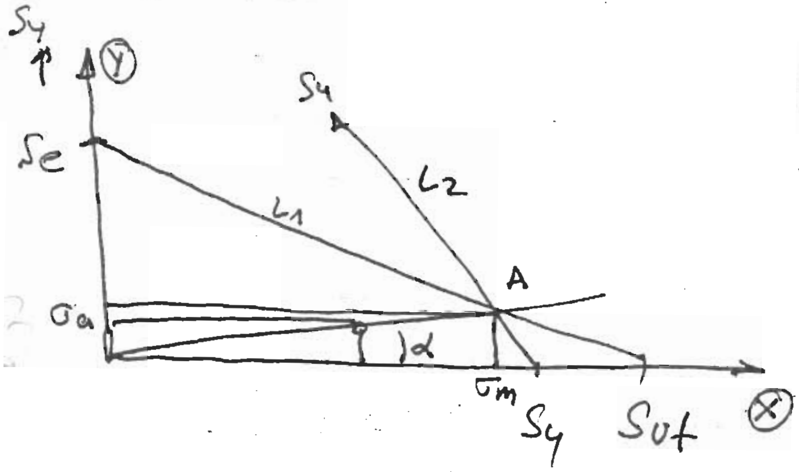
$$\tau_t = \frac{32T}{\pi d^4} = \frac{S_{o}t}{2}$$

$$\frac{F_s = \frac{2T}{d}}{\xi \cdot s \cdot d \cdot l} = \frac{32T}{\pi d^4}$$

$$\frac{\cancel{2T}}{\xi \cdot s \cdot \cancel{d^2} \cdot l_2} = \frac{\overset{16}{\cancel{32}} \cancel{T}}{\pi \cancel{d^4}^{\,2}}$$

$$\boxed{l = \frac{\pi d^2}{16 \xi s}}$$

Solution P6 #5 - 15 marks



$$\begin{cases} \dfrac{X}{S_{ut}} + \dfrac{Y}{S_e} = 1 & (L_1) \\ \dfrac{X}{S_y} + \dfrac{Y}{S_y} = 1 & (L_2) \end{cases}$$

2

the intersection of lines L_1 and L_2 is point A

$$Y = S_y\left(1 - \frac{X}{S_y}\right) = S_y - X \qquad (L_2)$$

2

$$\frac{X}{S_{ut}} + \frac{S_y - \cancel{X}}{S_e} = 1 \qquad (L_1)$$

4

$$X S_e + S_{ut} \cdot S_y - X S_{ut} = S_e S_{ut}$$

$$X(S_{ut} - S_e) = S_{ut}(S_y - S_e)$$

$$X = S_{ut}\,\frac{S_y - S_e}{S_{ut} - S_e}$$

12

$$Y = S_y - S_{ut}\,\frac{S_y - S_e}{S_{ut} - S_e} = \frac{S_y \cancel{S_{ut}} - S_y S_e - \cancel{S_{ut} S_y} + S_{ut} S_e}{S_{ut} - S_e}$$

2

$$Y = \frac{S_e(S_{ut} - S_y)}{S_{ut} - S_e}$$

$\tan\alpha$: for any ratio $\dfrac{\sigma_a}{\sigma_m} \geq \dfrac{\dfrac{S_e(S_{ut} - S_y)}{\cancel{S_{ut} - S_e}}}{S_{ut}\,\dfrac{S_y - S_e}{\cancel{S_{ut} - S_e}}} = \dfrac{S_e}{S_{ut}}\,\dfrac{S_{ut}}{S_y -}$

$$R = \frac{S_e}{S_{ut}} \cdot \frac{S_{ut} - S_y}{S_y - S_e}$$

for the given values

$$R = \frac{0.4}{1} \cdot \frac{1.0 - 0.8}{0.8 - 0.4} = \frac{0.4 \times 0.2}{0.4} = 0.2$$

$\frac{\sigma_a}{\sigma_m} > 0.2$, slightly larger the safety factor is.