FACULTY OF ARTS AND SCIENCE
University of Toronto

MAT235Y - Calculus II
Summer 2012

Midterm Exam, June 28, 2012

Examiners: N. Hoell, E. Mazzeo, G. Richards, J. Watts
Duration: 1 hour and 50 Minutes

NO AIDS ALLOWED. **Total: 100 marks**

Family Name: ____________________
(Please Print)

Given Name(s): ____________________
(Please Print)

Lecture section (morning or evening): ____________________
(Please Print)

Student ID Number: ____________________

FOR MARKER'S USE ONLY			
Problem 1:	/20	Problem 2:	/15
Problem 3:	/15	Problem 4:	/15
Problem 5:	/15	Problem 6:	20
		TOTAL:	/100

Useful formulae:

Double angle formulae: $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$, $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$.

1. (20 points) For each of the following functions: (i) state the domain of the function, (ii) state where the function is continuous.

NOTE: You do not need to prove that a function is continuous wherever you claim it is. However, if you claim that a function is **not** continuous at a point, this should be justified (for example, using the two-path test).

(a) $f(x, y, z) = e^{-xy} \sin\left(\frac{\pi z}{2}\right)$,

(b) $f(x, y) = \begin{cases} \frac{x}{\sqrt{x^2 - y^2}} & \text{if } (x, y) \neq (0, 0), \\ 1 & \text{if } (x, y) = (0, 0). \end{cases}$

2. (15 points) Find a vector function that parameterizes the curve of intersection of the cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

3. (15 points) The temperature at a point (x, y, z) is given by $T(x, y, z) = 200e^{-x^2-3y^2-9z^2}$, where T is measured in °C and x, y, z in meters.

(a) Find the rate of change of temperature at the point $P(2, -1, 2)$ in the direction toward the point $(3, -3, 3)$.

(b) From the point P, in which direction is the temperature decreasing fastest?

4. (15 points) If $z = f(x, y)$ where $x = r\cos(\theta)$ and $y = r\sin(\theta)$,

(a) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ (in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$).

(b) Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

5. (15 points) Classify all local extremes of the function $f(x, y) = (x^2 + y^2)e^{y^2 - x^2}$.

6. (20 points) Determine the absolute maximum and minimum values of $f(x, y) = x - 1 - \frac{1}{3}(x - 1)^3 + y^2$ on the disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$.

(*scratch paper*)