5. Which of the following propositions have probability 1? a-c

(a) Av~A
(b) ~(A&~A)
(c) [(A&~B) v (A&B)] v [(~A&B)v(~A&~B)]
(d) (A&B) v (A&~B) v ~B
(e) none of the above

6. You are Jack's doctor, and think Jack has a 15% chance of having H1N1, and tested his blood sample. The result is *positive* (meaning that it says that he's got H1N1). The accuracy rate of the test is 80% . What would you conclude? d

(a) Jack has an 80% chance of having H1N1.
(b) Jack is more likely than not to have H1N1, but the chance is less than 80%.
(c) Jack has a 15% chance of having H1N1.
(d) It is more likely than not that Jack does not have H1N1.
(e) None of the above.

7. The situation is as in problem #6. What would you think is the probability that Jack has H1N1 given the test result?

$0.15x0.8 / [0.15x0.8 + 0.85x0.2] = 3x4 / [3x4 + 17x2] = 12/29$

8. You are Jack's doctor, and think Jack has a 20% chance of having H1N1, and tested his blood sample. The result is *positive*, and the accuracy rate of the test is 80%. What would you conclude?
e
(a) Jack has an 80% chance of having H1N1.
(b) Jack is more likely than not to have H1N1, but the chance is less than 80%.
(c) Jack has a 15% chance of having H1N1.
(d) It is more likely than not that Jack does not have H1N1.
(e) None of the above.

9. You are Jack's doctor, and think Jack has 80% chance of having H1N1, and send his blood samples to 3 labs. They subjected the samples to the same test, which has 80% accuracy rate. And two of the labs reported positive test results while one of them reported negative test result. What would you conclude?

(a) The results are worthless.
(b) It is more likely than not that Jack does *not* have H1N1.
(c) It is slightly more than not that Jack has H1N1.
(d) It is much more likely than not that Jack has H1N1.
(e) None of the above.

10. The situation is as in Problem #9. What would you think is the probability that Jack has H1N1 given the test result?

$$\frac{\cancel{8}\times 8\phi^2\times\cancel{2}\phi}{\cancel{8}\times 8\phi^2\times\cancel{2}\phi+\cancel{2}\times 2\phi^2\times\cancel{8}\phi}=\frac{8^2}{8^2+2^2}=\frac{4^2}{4^2+1}=\frac{16}{17}$$

Part II. [10 points each] *Calculations*: It is not necessary to give any justification or give your calculation.

11-13. Answer the following questions about the situation given below:
There are two urns, A and B. Urn A has 8 red balls and 2 green balls; Urn B has 2 red balls and 8 green balls. (You cannot see the balls inside the urn.) Now, you toss a fair coin. If it falls heads, you pick 1 ball from Urn A, do *not* put it back in, and pick another ball from the same urn. If the coin falls tails, you do the same with the balls in Urn B.

11. Are you more likely to pick a red ball than a green ball in the second drawing? No.

12. What is the probability of picking a red ball in the second drawing? 1/2.

13. What is the probability of picking red balls in both drawings, given that you pick a red ball in the second? 29/45.

$$P(R_1\&R_2)=\frac{1}{2}\cdot\left(\frac{8}{10}\right)\cdot\left(\frac{7}{9}\right)+\frac{1}{2}\left(\frac{2}{10}\right)\cdot\frac{1}{9}.$$

$$P(R_1\&R_2/R_1)=\left(\frac{8}{10}\right)\cdot\left(\frac{7}{9}\right)+\frac{2}{10}\cdot\frac{1}{9}=\frac{58}{90}=\frac{58}{90}=\frac{29}{45}.$$

600−200
−100 1/4
1/2 · 1/2

14. Martin is an illegal vendor. On each day of work he sells \$600 on average, but the cost for the products he sells is \$200. And he gets tickets of \$100 once in 4 days on the average. If he gets a ticket, he waits until he is charged in court, goes to the court to challenge the ticket, and pays the fine only if he loses. The court calls half of the times when he gets the ticket, and he wins half of the times when he goes to the court. What is the expected value of a day's work for Martin?

$$Exp(W) = (600-200) - 100 \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2}$$
$$= 400 - \frac{25}{4} = 400 - 6 \cdot \frac{1}{4} = 393 \cdot \frac{3}{4}.$$

15. Fund-raisers for Sarah, a candidate for the presidency, sell 10 tickets, and they offer a lottery for those who buy the tickets. They will draw a winning ticket 2 times with replacement. (So the same ticket may win both times.) The prize for the first time is \$5000, and for the second \$10,000. Moreover, those who hold tickets that do not win in either drawing get the consolation prize of luncheon with Sarah that is worth \$100. Now, the fund-raisers request \$5,000 for a ticket. You are an economist who does not consider it an honor to contribute to Sarah's campaign or to have luncheon with her. If so,

15a. What is the expected value of getting the ticket at \$5,000 (for you)?
15b. What is the fair value of the ticket (for you)?

$$EV = FV - 5000 = 1581 - 5000 = 3419$$
$$FV = 5000 \times \frac{1}{10} + 10{,}000 \times \frac{1}{10} + 100\left(\frac{9}{10}\right)\left(\frac{9}{10}\right) = 1500 + 81 = 1581.$$

Part III [10 points each]. Give definitions of the following notions by completing the sentences.

16. Jointly exhaustive & mutually exclusive propositions

Propositions $A_1, A_2, \ldots, A_n$ are said to be *jointly exhaustive* if and only if .....

Propositions $A_1, A_2, \ldots, A_n$ are said to be *mutually exclusive* if and only if .....