ECO 370
Economics Of Organization
Test 2 – Thursday, August 1, 2013
1:10 PM to 3PM
Room 2130 – CCT Building

Students may use a non-programmable calculator. All work should be in pen written on the lined portions of the test booklets. Full marks will only be awarded to fully explained answers. Please read all questions carefully before answering them.

(15 arks) QUESTION 1:

(3 marks) (a) What hypothesis did Bertrand and Mullainathan set out to test in their study?

- **Bertrand and Mullainathan's earlier study studied the effect of takeover laws on executive pay – if the LCM model prevails – incentives are effected – but if the "skimming" (Hart model) applies – mean pay would be affected**
- **Because the earlier study did not resolve the question – the second study divided the sample between companies that had no dominant shareholder and those that did**

(3 marks) (b) Identify the two (2) groups of states Bertrand and Mullainathan used in the study?

- **One group of states involved hostile take-over legislation**
- **The other group involved no or mild take-over legislation**

(3 marks) (c) What is the relevance of the study design you identified in (b) to the objective of the study?

One would expect CEO pay to be lower in states with anti-takeover legislation?
One would expect CEO pay to be higher in states with anti-takeover legislation

(3 marks) (d) What result did Bertrand and Mullainathan find that supported the property rights model and the linear contract model?

Bertrand and Mullainathan concluded that, in response to the passage of anti-takeover legislation

- Mean pay rises as the Hart model would have predicted in firms without a dominant shareholder
- Performance pay rises as the LCM model would have predicted in firms with a dominant shareholder

(3 marks) (e) Identify the factors they found that accounted for the high level of executive compensation?

Some factors include:

the extent to which executives are driven by performance incentives to avoid potential moral hazard problems,

past successes of executives in maximizing firm wealth,

their control over compensation committee, and their sensitivity to public scrutiny.

(10 marks) QUESTION 2:

There are two firms, P and A. Firm A supplies a valuable input to Firm P. Each firm makes initial investments i_P and i_A. Each firm has revenue streams TR_P and TR_A independent of the productivity improvements that come from the investments. The profit functions are $\pi_P = TR_P + 2\alpha_P i_P^{1/2} - p - i_P$ and $\pi_A = p + TR_A + 2\alpha_A i_A^{1/2} - i_A$. The price p represents the contract price Firm P pays Firm A for one (1) unit of the input.

(4 marks) (a) **Suppose firms P and A are integrated. What levels of i_P and i_A create first best results for the integrated organization?**

First Best Solution involves joint Total Value (TV) maximization by both P and A:

$TV = \pi_P + \pi_A$

$TV = (TR_P + 2\alpha_P i_P^{1/2} - p - i_P) + (p + TR_A + 2\alpha_A i_A^{1/2} - i_A)$

$d(TV)/di_P = 0$

$d[(TR_P + \alpha_P i_P^{1/2} - p - i_P) + (p + TR_A + 2\alpha_A i_A^{1/2} - i_A)]/di_P = 0$

$d(TV)/di_P = 0 + \alpha_P i_P^{-1/2} - 0 - 1 + 0 = 0$

$\alpha_P i_P^{-1/2} = 1$

$\alpha_P = i_P^{1/2}$

$i_P = \alpha_P^2$

By symmetry $i_A = \alpha_A^2$

(3 marks) (b) **What is the total value (TV) of this organization?**

$TV = (TR_P + 2\alpha_P i_P^{1/2} - p - i_P) + (p + TR_A + 2\alpha_A i_A^{1/2} - i_A)$

$TV = TR_P + 2\alpha_P[\alpha_P^2]^{1/2} - [\alpha_P^2] + TR_A + 2\alpha_A[\alpha_A^2]^{1/2} - [\alpha_A^2]$

$TV = TR_P + 2\alpha_P\alpha_P - [\alpha_P^2] + TR_A + 2\alpha_A\alpha_A - [\alpha_A^2]$

$TV = TR_P + \alpha_P^2 + TR_A + \alpha_A^2$

(3 marks) (c) **"Firms should generally buy their inputs rather than make their inputs in order to avoid paying the costs necessary to make the output?" Please comment.**

This is only true if the input maker can exploit economies of scale and is a price taker. Even here, transaction costs could cancel cost savings.

(20 marks) QUESTION 3:

Suppose firms P and A are separate. The profit functions become $\pi_P = TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P$ and $\pi_A = \underline{p} + TR_A + 2\alpha^S{}_A i_A^{1/2} - i_A$. The price $\underline{p}$ represents the market price Firm P pays Firm A for one (1) unit of the

input. In all other respects the information set out in QUESTION 2 applies:

(5 marks) (a) **Derive the gains from trade for P and A if the firms move from the market to contract. Explain the function of the Theorem of Coase in generating the gains from trade. Assume the gains from trade will be divided equally between P and A.**

$$TV_{Integrated} = (TR_P + 2\alpha_P i_P^{1/2} - p - i_P) + (p + TR_A + 2\alpha_A i_A^{1/2} - i_A)$$

$$TV_{Separated} = (TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P) + (\underline{p} + TR_A + 2\alpha^S{}_A i_A^{1/2} - i_A)$$

$$TV_{Integrated} = TR_P + 2\alpha_P i_P^{1/2} - i_P + TR_A + 2\alpha_A i_A^{1/2} - i_A$$

$$TV_{Separated} = TR_P + 2\alpha^S{}_P i_P^{1/2} - i_P + TR_A + 2\alpha^S{}_A i_A^{1/2} - i_A$$

$$\text{Gains From Trade} = \Delta TV = TV_{Integrated} - TV_{Separated}$$

$$= (TR_P + 2\alpha_P i_P^{1/2} - i_P + TR_A + 2\alpha_A i_A^{1/2} - i_A) - (TR_P + 2\alpha^S{}_P i_P^{1/2} - i_P + TR_A + 2\alpha^S{}_A i_A^{1/2} - i_A)$$

$$= 2\alpha_P i_P^{1/2} + 2\alpha_A i_A^{1/2} - 2\alpha^S{}_P i_P^{1/2} - 2\alpha^S{}_A i_A^{1/2}$$

$$\Delta TV = 2(\alpha_P - \alpha^S{}_P) i_P^{1/2} + 2(\alpha_A - \alpha^S{}_A) i_A^{1/2}$$

(5 marks) (b) **Derive the price condition that relates contract price p to the market price $\underline{p}$.**

$$\pi_P = \pi^S{}_P + (1/2)\Delta TV \qquad \text{By Theorem of Coase}$$

$$TR_P + 2\alpha_P i_P^{1/2} - p - i_P = \pi^S{}_P + (1/2)\Delta TV$$

$$TR_P + 2\alpha_P i_P^{1/2} - p - i_P = TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P + (1/2)\Delta TV$$

$$2\alpha_P i_P^{1/2} - p = 2\alpha^S{}_P i_P^{1/2} - \underline{p} + (1/2)\Delta TV$$

$$2(\alpha_P - \alpha^S{}_P) i_P^{1/2} - p = -\underline{p} + (1/2)(2(\alpha_P - \alpha^S{}_P) i_P^{1/2} + 2(\alpha_A - \alpha^S{}_A) i_A^{1/2})$$

$$2(\alpha_P - \alpha^S{}_P) i_P^{1/2} - p = -\underline{p} + (\alpha_P - \alpha^S{}_P) i_P^{1/2} + (\alpha_A - \alpha^S{}_A) i_A^{1/2}$$

$$-p = -\underline{p} - (\alpha_P - \alpha^S{}_P) i_P^{1/2} + (\alpha_A - \alpha^S{}_A) i_A^{1/2}$$

$$p = \underline{p} + (\alpha_P - \alpha^S{}_P) i_P^{1/2} - (\alpha_A - \alpha^S{}_A) i_A^{1/2}$$

(5 marks) (c) What levels of i_P and i_A create second best results for the separate firms?

$$d(TR_P + 2\alpha^S{}_P i_P{}^{1/2} - \underline{p} - i_P + (1/2)\Delta TV)\ /di_P = 0$$
$$0 + \alpha^S{}_P i_P{}^{-1/2} - 0 - 1 + (1/2)(\alpha_P - \alpha^S{}_P)i_P{}^{-1/2} + 0 = 0$$
$$\alpha^S{}_P i_P{}^{-1/2} + (1/2)(\alpha_P - \alpha^S{}_P)i_P{}^{-1/2} = 1$$
$$\alpha^S{}_P + (1/2)(\alpha_P - \alpha^S{}_P) = i_P{}^{1/2}$$
$$(1/2)(\alpha_P + \alpha^S{}_P) = i_P{}^{1/2}$$
$$(1/4)(\alpha_P + \alpha^S{}_P)^2 = i_P$$

$$i_P = (1/4)(\alpha_P + \alpha^S{}_P)^2$$
$$i_A = (1/4)(\alpha_A + \alpha^S{}_A)^2 \qquad \text{by symmetry}$$

(5 marks) (d) What is the total value (TV) of this organization? What four (4) forms can asset specificity take?

$$TV_{Separated} = TR_P + 2\alpha^S{}_P i_P{}^{1/2} - i_P + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + 2\alpha^S{}_P[(1/4)(\alpha_P + \alpha^S{}_P)^2]^{1/2} - (1/4)(\alpha_P + \alpha^S{}_P)^2 + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + 2\alpha^S{}_P(1/2)(\alpha_P + \alpha^S{}_P) - (1/4)(\alpha_P + \alpha^S{}_P)^2 + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + \alpha^S{}_P(\alpha_P + \alpha^S{}_P) - (1/4)(\alpha_P + \alpha^S{}_P)^2 + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + (1/4)(4\alpha^S{}_P - \alpha_P - \alpha^S{}_P)(\alpha_P + \alpha^S{}_P) + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + (1/4)(3\alpha^S{}_P - \alpha_P)(\alpha_P + \alpha^S{}_P) + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$
$$TV_{Separated} = TR_P + TR_A + (1/4)(3\alpha^S{}_P - \alpha_P)(\alpha_P + \alpha^S{}_P) + (1/4)(3\alpha^S{}_A - \alpha_A)(\alpha_A + \alpha^S{}_A) \quad \text{by symmetry}$$

- Asset specificity 4 types:
  - » Site specificity
  - » Physical asset specificity
  - » Human asset specificity
  - » Dedicated assets

(15 marks) **QUESTION 4:**

Using the information from QUESTIONS 2 and 3, assume that P obtains β as its share of the <u>gains from trade</u> and A obtains $(1-\beta)$ as its share of the <u>gains from trade</u>:

(5 marks) (a) What levels of i_P and i_A create second best results for the separate firms?

$$d(TR_P + 2\alpha^S{}_P i_P{}^{1/2} - \underline{p} - i_P + \beta\Delta TV) / di_P = 0$$
$$0 + \alpha^S{}_P i_P{}^{-1/2} - 0 - 1 + (\beta/2)[2(\alpha_P - \alpha^S{}_P)] i_P{}^{-1/2} + 0 = 0$$
$$\alpha^S{}_P i_P{}^{-1/2} + \beta(\alpha_P - \alpha^S{}_P) i_P{}^{-1/2} = 1$$
$$\alpha^S{}_P + \beta(\alpha_P - \alpha^S{}_P) = i_P{}^{1/2}$$
$$\beta\alpha_P + (1 - \beta)\alpha^S{}_P = i_P{}^{1/2}$$
$$[\beta\alpha_P + (1 - \beta)\alpha^S{}_P]^2 = i_P$$

$$i_P = [\beta\alpha_P + (1 - \beta)\alpha^S{}_P]^2$$

$$d(TR_A + 2\alpha^S{}_A i_A{}^{1/2} + \underline{p} - i_A + (1 - \beta)\Delta TV) / di_A = 0$$
$$0 + \alpha^S{}_A i_A{}^{-1/2} - 0 - 1 + (1/2)(1 - \beta)[2(\alpha_A - \alpha^S{}_A)] i_A{}^{-1/2} + 0 = 0$$
$$\alpha^S{}_A i_A{}^{-1/2} + (1 - \beta)(\alpha_A - \alpha^S{}_A) i_A{}^{-1/2} = 1$$
$$\alpha^S{}_A + (1 - \beta)(\alpha_A - \alpha^S{}_A) = i_A{}^{1/2}$$
$$\alpha^S{}_A + \alpha_A - \alpha^S{}_A - \beta\alpha_A + \beta\alpha^S{}_A = i_A{}^{1/2}$$
$$\alpha_A - \beta\alpha_A + \beta\alpha^S{}_A = i_A{}^{1/2}$$
$$(1 - \beta)\alpha_A + \beta\alpha^S{}_A = i_A{}^{1/2}$$

$$I_A = [\beta\alpha^S{}_A + (1 - \beta)\alpha_A]^2$$

(5 marks) (b) What is the total value (TV) of this organization?

$$TV_{Separated} = TR_P + 2\alpha^S{}_P i_P{}^{1/2} - i_P + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$

$$TV_{Separated} = TR_P + 2\alpha^S{}_P[\beta\alpha_P + (1 - \beta)\alpha^S{}_P] - [\beta\alpha_P + (1 - \beta)\alpha^S{}_P]^2 + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$

$$TV_{Separated} = TR_P + [\beta\alpha_P + (1 - \beta)\alpha^S{}_P][2\alpha^S{}_P - \beta\alpha_P - \alpha^S{}_P + \beta\alpha^S{}_P] + TR_A + 2\alpha^S{}_A i_A{}^{1/2} - i_A$$

$$TV_{Separated} = TR_P + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + TR_A + 2\alpha^S_A i_A^{1/2} - i_A$$

$$TV_{Separated} = TR_P + TR_A + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + 2\alpha^S_A[\beta\alpha^S_A + (1 - \beta)\alpha_A] - [\beta\alpha^S_A + (1 - \beta)\alpha_A]^2$$

$$TV_{Separated} = TR_P + TR_A + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [2\alpha^S_A - \beta\alpha^S_A - (1 - \beta)\alpha_A][\beta\alpha^S_A + (1 - \beta)\alpha_A]$$

$$TV_{Separated} = TR_P + TR_A + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [(2 - \beta)\alpha^S_A - (1 - \beta)\alpha_A][\beta\alpha^S_A + (1 - \beta)\alpha_A]$$

$$TV_{Separated} = TR_P + TR_A + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [\alpha^S_A + (1 - \beta)\alpha^S_A - (1 - \beta)\alpha_A][\beta\alpha^S_A + (1 - \beta)\alpha_A]$$

$$TV_{Separated} = TR_P + TR_A + [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [\beta\alpha^S_A + (1 - \beta)\alpha_A][\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A)]$$

(5 marks) **(c) What is the optimal level of β?**

$$d[TV_{Separated}]/d\beta = 0$$

Let $A = [\beta\alpha_P + (1 - \beta)\alpha^S_P][\alpha^S_P - \beta(\alpha_P - \alpha^S_P)]$
Let $B = [\beta\alpha^S_A + (1 - \beta)\alpha_A][\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A)]$

$$d[TV_{Separated}]/d\beta = d[TR_P]/d\beta + d[TR_A]/d\beta + dA/d\beta + dB/d\beta = 0$$

$$d[TV_{Separated}]/d\beta = 0 + 0 + dA/d\beta + dB/d\beta = 0$$

$$d[TV_{Separated}]/d\beta = dA/d\beta + dB/d\beta = 0$$

$$dA/d\beta = (d[\beta\alpha_P + (1 - \beta)\alpha^S_P]/d\beta)[\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [\beta\alpha_P + (1 - \beta)\alpha^S_P](d[\alpha^S_P - \beta(\alpha_P - \alpha^S_P)/d\beta)]$$

$$dA/d\beta = (\alpha_P - \alpha^S_P)[\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] + [\beta\alpha_P + (1 - \beta)\alpha^S_P][-(\alpha_P - \alpha^S_P)]$$

$$dA/d\beta = (\alpha_P - \alpha^S_P)([\alpha^S_P - \beta(\alpha_P - \alpha^S_P)] - [\beta\alpha_P + (1 - \beta)\alpha^S_P])$$

$$dA/d\beta = (\alpha_P - \alpha^S_P)(\alpha^S_P - \beta\alpha_P + \beta\alpha^S_P - \beta\alpha_P - \alpha^S_P + \beta\alpha^S_P)$$

$$dA/d\beta = (-2\beta)(\alpha_P - \alpha^S_P)^2$$

$$dB/d\beta = (d[\beta\alpha^S_A + (1 - \beta)\alpha_A]/d\beta)[\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A)] + [\beta\alpha^S_A + (1 - \beta)\alpha_A](d[(\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A)]/d\beta)$$

$$dB/d\beta = (\alpha^S_A - \alpha_A)[\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A)] + (\alpha^S_A - \alpha_A)[\beta\alpha^S_A + (1 - \beta)\alpha_A]$$

$$dB/d\beta = (\alpha^S_A - \alpha_A)[\alpha^S_A - (1 - \beta)(\alpha_A - \alpha^S_A) + \beta\alpha^S_A + (1 - \beta)\alpha_A]$$

$$dB/d\beta = (\alpha^S_A - \alpha_A)[\alpha^S_A - \alpha_A + \alpha^S_A + \beta\alpha_A - \beta\alpha^S_A + \beta\alpha^S_A + \alpha_A - \beta\alpha_A]$$

$$dB/d\beta = (2\alpha^S_A)(\alpha^S_A - \alpha_A)$$

$$dA/d\beta + dB/d\beta = 0$$
$$(-2\beta)(\alpha_P - \alpha^S_P)^2 + (2\alpha^S_A)(\alpha^S_A - \alpha_A) = 0$$
$$\beta(\alpha_P - \alpha^S_P)^2 - (\alpha^S_A)(\alpha^S_A - \alpha_A) = 0$$

$$\beta = \alpha^S_A(\alpha^S_A - \alpha_A)/(\alpha_P - \alpha^S_P)^2$$

(15 marks) **QUESTION 5:**

Using the information from QUESTIONS 2 and 3, P takes over A:

(5 marks) (a) Derive the levels of i_P and i_A that P will invest.

$$TV_{Integrated} = \cancel{TR_P} + 2\alpha_P i_P^{1/2} - \cancel{i_P} + \cancel{TR_A} + 2\alpha_A i_A^{1/2} - \cancel{i_A}$$
$$TV_{Separated} = \cancel{TR_P} + 2\alpha^S{}_P i_P^{1/2} - \cancel{i_P} + \cancel{TR_A} + \cancel{2\alpha^S{}_A i_A^{1/2}} - \cancel{i_A}$$

$$\Delta TV_{UI(BI)} = 2(\alpha_P - \alpha^S{}_P) i_P^{1/2} + 2\alpha_A i_A^{1/2}$$
$$(1/2)\Delta TV_{UI(BI)} = (\alpha_P - \alpha^S{}_P) i_P^{1/2} + \alpha_A i_A^{1/2}$$

$$d(TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P - i_A + (1/2)\Delta TV)\ /di_P = 0$$
$$d(TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P - i_A + (1/2)[(\alpha_P - \alpha^S{}_P) i_P^{1/2} + \alpha_A i_A^{1/2}])/di_P = 0$$

$$0 + \alpha^S{}_P i_P^{-1/2} - 0 - 1 + (1/2)(\alpha_P - \alpha^S{}_P) i_P^{-1/2} + 0 = 0$$
$$\alpha^S{}_P i_P^{-1/2} + (1/2)(\alpha_P - \alpha^S{}_P) i_P^{-1/2} = 1$$
$$\alpha^S{}_P + (1/2)(\alpha_P - \alpha^S{}_P) = i_P^{1/2}$$
$$(1/2)(\alpha_P + \alpha^S{}_P) = i_P^{1/2}$$
$$(1/4)(\alpha_P + \alpha^S{}_P)^2 = i_P$$

$$i_P = (1/4)(\alpha_P + \alpha^S{}_P)^2$$

$$d(TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P - i_A + (1/2)\Delta TV)\ /di_A = 0$$
$$d(TR_P + 2\alpha^S{}_P i_P^{1/2} - \underline{p} - i_P - i_A + (1/2)[(\alpha_P - \alpha^S{}_P) i_P^{1/2} + \alpha_A i_A^{1/2}])/di_A = 0$$

$$(1/2)\alpha_A i_A^{-1/2} = 1$$
$$(1/2)\alpha_A = i_A^{1/2}$$

$$i_A = (1/4)\alpha_A^2$$

(5 marks) (b) What is the total value (TV) of this organization?

$$TV_{UI(BI)} = TR_P + 2\alpha^S{}_P i_P^{1/2} - i_P + TR_A + 2\alpha^S{}_A i_A^{1/2} - i_A$$

$$TV_{UI(BI)} = TR_P + 2\alpha^S{}_P[(1/4)(\alpha_P + \alpha^S{}_P)^2]^{1/2} - [(1/4)(\alpha_P + \alpha^S{}_P)^2] + TR_A + 2\alpha_A[(1/4)\alpha_A^2]^{1/2} - (1/4)\alpha_A^2$$

$$TV_{UI(BI)} = TR_P + \alpha^S{}_P(\alpha_P + \alpha^S{}_P) - (1/4)(\alpha_P + \alpha^S{}_P)^2 + TR_A + \alpha_A\alpha_A - (1/4)\alpha_A^2$$

$TV_{UI(BI)} = TR_P + [\alpha^S_P - (1/4)(\alpha_P + \alpha^S_P)](\alpha_P + \alpha^S_P) + TR_A + (3/4)\alpha_A^2$

$TV_{UI(BI)} = TR_P + TR_A + (1/4)[3\alpha^S_P - \alpha_P](\alpha_P + \alpha^S_P) + (3\alpha_A^2)/4$

(5 marks) (c) Explain the significance of this take-over by relating it to the ideas of "residual rights of control", "residual claimant" and the "residual return".

Due to the takeover, firm P ends up owning both assets of P and A giving it the <u>residual rights of control</u> over the assets. This is the right to make any decisions not explicitly referred to in the contract, concerning asset use across the vertical supply chain.
[Milgrom and Roberts – 289]
As residual claimant, firm P will control the residual return generated by the assets.
[Milgrom and Roberts – 291]

(10 marks) QUESTION 6:

Using the information in QUESTIONS 2 and 3, suppose the investment level of P is completely contractible – that is both observable and enforceable:

(5 marks) (a) Derive the levels of i_P and i_A that P and A will invest.

$d(TV)/di_P = 0$

$d[(TR_P + \alpha_P i_P^{1/2} - p - i_P) + (p + TR_A + 2\alpha_A i_A^{1/2} - i_A)]/di_P = 0$

$d(TV)/di_P = 0 + \alpha_P i_P^{-1/2} - 0 - 1 + 0 = 0$

$\alpha_P i_P^{-1/2} = 1$

$i_P = \alpha_P^2$

(5 marks) (b) How would A choose the type of organization most optimal for P and A?

Firm A strategically assesses all the organizational options by estimating TV for each choice, subject to the constraint $i_P = \alpha_P^2$, then Firm A chooses the choice that maximizes TV.

(15 marks) **QUESTION 7:**

There are two risk neutral firms, P and A. Firm A supplies two (2) inputs X_1 and X_2 to P. The profit function for A is $\pi_A = \beta(\gamma_1 X_1 + \gamma_2 X_2) - (1/2)(X_1^2 + X_2^2)$. The profit function for P is $\pi_P = (\delta_1 + \kappa_1)X_1 + (\delta_2 + \kappa_2)X_2 - \beta(\gamma_1 X_1 + \gamma_2 X_2)$.

(3 marks) (a) Derive the value of β if P and A make output $Y = \delta_1 X_1 + \delta_2 X_2$ within an integrated organization. Output price is 1.

$$\pi_A = \beta(\gamma_1 X_1 + \gamma_2 X_2) - (1/2)(X_1^2 + X_2^2)$$

The following are the income compatibility constraints for firm A:

$$d\pi_A / dX_1 = 0 \text{ implies } X_1 = \beta\gamma_1$$
$$d\pi_A / dX_2 = 0 \text{ implies } X_2 = \beta\gamma_2$$

$$TV = \pi_P + \pi_A$$
$$TV = (\delta_1 + \kappa_1)X_1 + (\delta_2 + \kappa_2)X_2 - (1/2)(X_1^2 + X_2^2)$$
$$TV = (\delta_1 + \kappa_1)(\beta\gamma_1) + (\delta_2 + \kappa_2)(\beta\gamma_2) - (1/2)(\beta^2\gamma_1^2 + \beta^2\gamma_2^2)$$

$$d(TV)/d\beta = (\delta_1 + \kappa_1)(\gamma_1) + (\delta_2 + \kappa_2)(\gamma_2) - (\beta\gamma_1^2 + \beta\gamma_2^2) = 0$$

$$\beta_{MAKE} = [(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/(\gamma_1^2 + \gamma_2^2)$$

(3 marks) (b) Derive the total value (TV) for this organization.

$$TV_{MAKE} = \beta[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2] - (\beta^2/2)(\gamma_1^2 + \gamma_2^2)$$

$$TV_{MAKE} = [[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/(\gamma_1^2 + \gamma_2^2)][(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2] - (1/2)[[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/(\gamma_1^2 + \gamma_2^2)]^2(\gamma_1^2 + \gamma_2^2)$$

$$TV_{MAKE} = [(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]^2/(\gamma_1^2 + \gamma_2^2)] - (1/2)[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]^2/(\gamma_1^2 + \gamma_2^2)$$

$$TV_{MAKE} = (1/2)[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]^2/(\gamma_1^2 + \gamma_2^2)$$

(3 marks) (c) **Derive the value of β if P buys the inputs from A where the profit function for A is $\pi_A = \beta(\gamma_1 X_1 + \gamma_2 X_2) + \kappa_1 X_1 + \kappa_2 X_2 - (1/2)(X_1^2 + X_2^2)$ and the profit function for P is $\pi_P = \delta_1 X_1 + \delta_2 X_2 - \beta(\gamma_1 X_1 + \gamma_2 X_2)$.**

$$\pi_A = \beta(\gamma_1 X_1 + \gamma_2 X_2) + \kappa_1 X_1 + \kappa_2 X_2 - (1/2)(X_1^2 + X_2^2)$$

The following are the income compatibility constraints for firm A:

$d\pi_A/dX_1 = 0$ implies $X_1 = \beta\gamma_1 + \kappa_1$
$d\pi_A/dX_2 = 0$ implies $X_2 = \beta\gamma_2 + \kappa_2$

$$TV = \pi_P + \pi_A$$
$$TV = \delta_1 X_1 + \delta_2 X_2 + \kappa_1 X_1 + \kappa_2 X_2 - (1/2)(X_1^2 + X_2^2)$$
$$TV = (\delta_1 + \kappa_1)X_1 + (\delta_2 + \kappa_2)X_2 - (1/2)(X_1^2 + X_2^2)$$

$$TV = (\delta_1 + \kappa_1)(\beta\gamma_1 + \kappa_1) + (\delta_2 + \kappa_2)(\beta\gamma_2 + \kappa_2) - (1/2)([\beta\gamma_1 + \kappa_1]^2 + [\beta\gamma_2 + \kappa_2]^2)$$

$$d(TV)/d\beta = (\delta_1 + \kappa_1)(\gamma_1) + (\delta_2 + \kappa_2)(\gamma_2) - \gamma_1[\beta\gamma_1 + \kappa_1] - \gamma_2[\beta\gamma_2 + \kappa_2] = 0$$

$$\delta_1\gamma_1 + \delta_2\gamma_2 - \beta[\gamma_1^2 + \gamma_2^2] = 0$$
$$\beta_{BUY} = [\delta_1\gamma_1 + \delta_2\gamma_2]/[\gamma_1^2 + \gamma_2^2]$$

(3 marks) (d) **Derive the total value (TV) for this organization.**

$$TV_{BUY} = \beta_{BUY}[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2] - ([\beta_{BUY}]^2/2)(\gamma_1^2 + \gamma_2^2)$$
$$TV_{BUY} = [\delta_1\gamma_1 + \delta_2\gamma_2][(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/[\gamma_1^2 + \gamma_2^2] - ([\delta_1\gamma_1 + \delta_2\gamma_2]^2/2[\gamma_1^2 + \gamma_2^2]^2)(\gamma_1^2 + \gamma_2^2)$$
$$TV_{BUY} = [\delta_1\gamma_1 + \delta_2\gamma_2][(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/[\gamma_1^2 + \gamma_2^2] - (1/2)[\delta_1\gamma_1 + \delta_2\gamma_2]^2/(\gamma_1^2 + \gamma_2^2)$$

(3 marks) (e) What conclusions could one draw from these derivations concerning the make or buy decision.

$\beta_{MAKE} = [(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/(\gamma_1^2 + \gamma_2^2) > \beta_{BUY} = [\delta_1\gamma_1 + \delta_2\gamma_2]/[\gamma_1^2 + \gamma_2^2]$ **suggesting that higher incentives should be awarded in integrated organizations**

$TV_{BUY} = [\delta_1\gamma_1 + \delta_2\gamma_2][(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]/[\gamma_1^2 + \gamma_2^2] - (1/2)[\delta_1\gamma_1 + \delta_2\gamma_2]^2/(\gamma_1^2 + \gamma_2^2) > TV_{MAKE} = (1/2)[(\delta_1 + \kappa_1)\gamma_1 + (\delta_2 + \kappa_2)\gamma_2]^2/(\gamma_1^2 + \gamma_2^2)$ **suggesting that buy decisions should happen when the market can accommodate scale and standardization advantages.**