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UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO206Y1Y (Microeconomic Theory)
Instructor: Victor Couture and Rebecca Lindstrom

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Duration: 180 minutes (3 hours)

Examinations Aids: Non-Programmable Calculators

This examination paper consists of **16** pages and **8** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

To obtain credit, you must give arguments to support your answers.

For graders' use:

	Score
1 (13)	
2 (9)	
3 (10)	
4 (10)	
5 (21)	
6 (17)	
7 (10)	
8 (10)	
Total (100)	

1. Lude has \$12,000. He plans to bet on a football game. Assume no ties can occur. Experts think that team A is more likely to win, and the odds at the local gambling house are 3 to 1 in favor of team A, so that for \$0.25 one can buy or sell a ticket that pays \$1 if team B wins and nothing if A wins. Lude, however, thinks that the two teams are equally likely to win. Lude is an expected utility maximizer, and his utility when he has a wealth of w is $ln(w)$.

   (a) [3] Find an expression for Lude's budget constraint.

   Solution: There are two state of nature: team A wins (state A) and team B wins (state B). Let w_A be Lude's wealth if team A wins, w_B be his wealth if team B wins and T be the number of tickets that he purchase. We have that $w_A = 12000 - 0.25T$ and $w_B = 12000 - 0.25T + T$. Isolating $T = (12000 - w_A)/0.25$ in the expression for w_A and plugging it into the expression for w_B we obtain the budget constraint $3w_A + w_B = 48000$.

   (b) [2] Find an expression for Lude's expected utility function.

   Solution: Lude believes that team A wins with probability 0.5 and that team B wins with probability 0.5, so his expected utility is $0.5ln(w_A) + 0.5ln(w_B)$.

   (c) [4] How many tickets does Lude buy or sell?

   Solution: At an optimum we have that $MU_A/MU_B = p_A/p_B$, which we can write as $\frac{0.5w_B}{0.5w_A} = \frac{3}{1}$ and solve for $w_B = 3w_A$. Plugging w_B in the budget constraint we can find $w_A = 48000/6 = 8000$. Then using our expression for w_A in part a) we solve $8000 = 12000 - 0.25T$ to find that Lude buys $T = 16000$ tickets.

   (d) [4] After Lude finishes his transaction at the gambling house, team A loses a star player and the ticket price moves to \$0.40. Lude can buy new tickets or sell those he already has. He now believes that team A has a 30% chance of winning. Suppose that when the game is finally played, team A wins. How much wealth does he end up with?

   Solution: Lude bought 16000 tickets, which cost him \$4000, so he has $12000 - 4000 = 8000$ dollars left. The value of his tickets is now $16000 * 0.4 = 6400$, so his new wealth is $8000 + 6400 = 14400$. We can find the budget constraint as before. We have that $w_A = 14400 - 0.4T$ and $w_B = 14400 - 0.4T + T$. Isolating $T = (14400 - w_A)/0.4$ in the expression for w_A and plugging it into the expression for w_B we obtain the budget constraint $(3/2)w_A + w_B = (5/2)14400$. Lude's expected utility is now $0.3ln(w_A) + 0.7ln(w_B)$. At an optimum we have that $MU_A/MU_B = p_A/p_B$, which we can write as $\frac{0.3w_B}{0.7w_A} = \frac{3/2}{1}$ and solve for $w_B = (7/2)w_A$. Plugging w_B in the budget constraint we can find $(3/2)w_A + (7/2)w_A = (5/2)14400$ and solve it for $w_A = 7200$, which is Lude's wealth team A wins.

2. A competitive firm produces a single output using a single input. In 2010, the cost of the input was \$1 and the price of the output was \$6, and the firm used 3 units of

input to produce 2 units of output. In 2011, the cost of the input was \$3 and the price of the output was \$10, and the firm used 1 unit of input to produce 1 unit of output. The firm's technology is the same in 2010 and in 2011.

(a) [3] With this information only, can you tell whether the firm was profit-maximizing in 2010?

Solution: If a firm is profit-maximizing in 2010, then at 2010 prices its 2010 input and output choices must lead to a higher profit than its 2011 input and output choices, i.e. we must have that $6(2) - 1(3) = 9$ is larger than $6(1) - 1(1) = 5$ (which is true). So its 2010 choice is consistent with profit-maximization, but we cannot tell whether it is profit-maximizing (so the answer is that we cannot tell). Note that it possible to provide a perfectly valid graphical argument to arrive at the same conclusion.

(b) [3] With this information only, can you tell whether the firm is profit-maximizing in 2011?

Solution: If the firm is profit-maximizing in 2011, then at 2011 prices its 2011 input and output choices must lead to a higher profit than its 2010 input and output choices, i.e. we must have that $10(1) - 3(1) = 7$ is larger than $10(2) - 3(3) = 11$ (which is not true). So the firm's 2011 choice is not consistent with profit-maximization, and we can tell that the firm is not profit-maximizing in 2011. Again, it is also possible to provide a perfectly valid graphical argument.

(c) [3] Suppose that you are told that the firm's technology exhibits increasing returns to scale (IRS). Now, can you tell whether the firm was profit-maximizing in 2010? Explain carefully.

Solution: With increasing returns to scale, the profit-maximizing problem has not solution (and therefore a firm with IRS cannot be profit-maximizing); profits can be increased indefinitely just by increasing the scale of production. To show this, consider a profit function like $\pi = pf(y) - wx$. By scaling up its operation by a factor t, the value of the firm's production ($pf(tx)$) will increase by a factor larger than t ($pf(tx) > tpf(x), \forall t > 1$, by definition of IRS), and its cost ($wtx = twx$) will increase by a factor t. So a firm with IRS technology can always increase its profits by increasing the scale of production, and therefore the 2010 choice of $y = 2$ and $x = 3$ cannot be profit-maximizing.

3. Firm Z uses two factors to produce airplanes using the technology

$$y = \min\{x_1, 2x_2\}$$

where y is the number of airplanes, and x_1 and x_2 are the quantities of factor 1 (airplane bodies) and factor 2 (airplane wings), respectively. The two factors are bought in competitive factors markets where the price of factor 1 is 9 per unit and the price of factor 2 is 5 per unit. Because firm Z has limited warehouse space, it cannot use more than 10 units of factor 1 in production.

If firm Z produces some positive number of airplanes ($y > 0$), then it incurs an additional cost of A where $A \geq 0$. If firm Z produces no airplanes ($y = 0$), it does not incur this additional cost, i.e., the total cost is zero.

(a) [5] Suppose the market for airplanes is competitive with a very large number of firms, all choosing to supply some $y > 0$. These firms are all profit maximizers and have the same technology (and warehouse constraints) as firm Z. Let $A = 90$. In the long run, what is the price of airplanes?

Solution: In the long run, firms enter if they can make positive profit, and exit if they make negative profit. Note that here, since there are no truly fixed costs, producing $y = 0$ (being "inactive") and exiting are equivalent, so they can be used interchangeably.
Suppose there is a LR equilibrium with price p where some firm is active but chooses some $y < 10$ (in other words, $y \in (0, 10)$). For this range of y, the marginal cost is constant at 19. From the fact that the firm does not exit (or equivalently, chooses $y > 0$), we know that

$$\pi = (p - 19)y - 90 \geq 0 \quad \Rightarrow \quad p \geq \frac{90}{y} + 19 > 19 = MC$$

But since price is above the constant marginal cost, it is easy to see that the firm can increase profit by expanding output. This is a standard IRS argument. Hence, choosing $y \in (0, 10)$ cannot maximize the firm's profit. Hence, no profit-maximizing firm will choose $y \in (0, 10)$ in LR eqm.
Following the argument above, all firms that produce something at a given price p in a LR eqm must choose $y = 10$. If firms neither exit nor enter (in LR eqm) we assume that

$$\pi = (p - 19)10 - 90 = 0 \quad \Rightarrow \quad p = 9 + 19 = 28$$

(although this equality holds only approximately in general, we can assume it holds generally since the number of firms is "very large"). To see that this is indeed profit-maximizing for the firm (this is not necessary since we know that some firms are active but that they cannot have chosen $y \in (0, 10)$), note that if the firm reduces output by some amount Δ, then the fall in cost is 19Δ, while the fall in revenue is 28Δ. Thus, this would reduce the firm's profit (making it negative rather than zero). Hence, choosing $y = 10$ (and making zero profit) is indeed profit-maximizing for the firm. Hence, $p = 28$ must be the LR equilibrium price.

Marks distribution

- Give [2] for calculating $MC = 19$, even if don't state for what range of y
- Give [2] for arguing that firms will not choose $y \in (0, 10)$ in LR eqm
- Give [1] for solving for $p = 28$ [1]

(b) [5] Now instead suppose that firm Z is a profit-maximizing monopolist in the market for airplanes. Let $A = 0$. Graphically, illustrate the extent of social welfare loss from monopoly pricing (deadweight loss) in this market. You should assume that market demand is decreasing in price.

Solution: It is necessary to consider two cases (ideally, drawing two graphs – or a messy graph with two sets of MR curves in).
Case 1: If the MR curve cuts the MC curve at $y < 10$, then we have the usual constant MC with positive ("triangle" shaped) DWL.
Case 2: If the MR curve cuts the $p = 19$ line to the right of $y = 10$, then the monopolist will choose to supply $y = 10$. This is the efficient level of output (no quantity distortions). Hence, there is no DWL. The only effect is the price increase (which will now be above MC).

Marks distribution

- Give [2] for Case 1: must draw constant MC and triangular DWL. Give only [1] if draw MC as increasing rather than constant.
- Give [3] for Case 2: state MR conditions for this case to occur [1], argue no DWL [1] because quantity does not change compared to efficient benchmark [1].

4. Be-For-Tea is the only tea producer in Atlantis, a closed island economy in the West Pacific. Be-For-Tea is a profit maximizing firm and can produce tea at a constant marginal cost of 20 dollars per pound.

Atlantis has two provinces: North and South. In the richer North, the inverse market demand for tea is given by

$$P_N(y) = 100 - y, \qquad 0 \leq y \leq 100$$

where P is the price of tea in dollars per pound and y is the amount of tea (in pounds). In the poorer South, the inverse market demand for tea is given by

$$P_S(y) = 40 - 2y, \qquad 0 \leq y \leq 20$$

where P and y are defined similarly. Although Be-For-Tea does not know each consumer's *individual* demand, it does know the market demand in each of the provinces as given above.

(a) [4] Suppose it is impossible for consumers to trade tea across the North-South provincial border. Write down Be-For-Tea's profit maximization problem and calculate its total profit from tea sales in Atlantis.

Solution: Since there is no possibility for consumer arbitrage, the monopolist can charge different prices in different markets (i.e., market segmentation/3rd degree price discrimination). The problem is given by

$$\max_y \{(a - by)y - 20y \;:\; y \geq 0\}$$

so we have $y^* = \dfrac{a-20}{2b}$ with

$$y_N^* = \frac{100-20}{2} = 40, \qquad p_N^* = 60, \qquad \pi_N^* = (60-20)40 = 1,600$$
$$y_S^* = \frac{40-20}{2(2)} = 5, \qquad p_S^* = 30, \qquad \pi_S^* = (30-20)5 = 50$$

Total profit is 1,650.

Marks distribution

- Give [1] for realizing (implicitly or explicitly) that the firm chooses market (province) specific prices
- Give [2] for writing down a correct profit maximization problem and taking correct FOC (don't penalize if fail to state non-negativity constraint on y). Since MC is constant, can write either one or two profit-maximization problems (equivalent).
- Give [2] for each correct profit/sum of profits. Don't penalize if they fail to add them up to the total profit.

(b) [6] Suppose consumers can trade tea freely across the North-South provincial border. Write down Be-For-Tea's profit maximization problem and calculate its total profit from tea sales in Atlantis.

Solution: Here, the monopolist should set the same price in both markets. Otherwise, purchases would ever only be made in the market with the lower price. First, we need to find the market demand curve. Invert the demand functions so that we can sum them. Then, we get

$$y_N(P) = 100 - p, \qquad 0 \le p \le 100$$
$$y_S(P) = 20 - \frac{1}{2}p, \qquad 0 \le p \le 40$$

The market demand is given by

$$y(P) = \begin{cases} 120 - \frac{3}{2}p & 0 \le p \le 40 \\ 100 - p, & 40 < p \le 100 \end{cases} \quad \Leftrightarrow \quad P(y) = \begin{cases} 100 - y & 0 \le y \le 60 \\ 80 - \frac{2}{3}y & 60 \le y \le 100 \end{cases}$$

Note that it is continuous but kinked, so the MR curve is discontinuous. If you draw the graph, it is obvious that the monopolist wants to price on the top piece (serving only consumers in the North) because the MR is always negative on the piece where both markets are served.
To see this formally, note that the marginal revenue curve is

$$MR(y) = \begin{cases} 100 - 2y & 0 \le y \le 60 \\ 80 - \frac{4}{3}y & 60 \le y \le 100 \end{cases}$$

An elasticity argument will work as well (this part is always inelastic). But for $y \ge 60$, $MR(y) = 80 - \frac{4}{3}y \le 80 - \frac{4}{3}60 = 0$, so the monopolist never prices on this piece of the demand curve. Hence, we can write the monopolist's problem

$$\max_y \{(100-y)y - 20y \ : \ y \geq 0\}$$

$$y^* = \frac{100-20}{2} = 40, \qquad p^* = 60, \qquad \pi^* = (60-20)40 = 1,600$$

This is the same as that for the North only above. It's easy to confirm $0 \leq y^* \leq 60$, so we are indeed on the part of the demand curve where both markets are served.

Marks distribution

- Give [1] for realizing (implicitly or explicitly) that the firm is constrained to choosing a single price
- Give [1] for correct market supply / inverse market supply
- Give [2] for arguing that will not supply on the part of demand curve where south are served (graphically or algebraically); an elasticity argument could work as well
- Give [1] for calculating total profit

Note: If students don't pay attention to the fact that the demand curve is kinked, they may want to set $80 - \frac{4}{3}y = 20 \Rightarrow y^* = 45$. This is incorrect. In this case, give at most [1] (for the first point on the marking scheme).

5. Kai and Lina are playing a two-player, two-action game with simultaneous moves represented in the following payoff matrix

		Lina: T	Lina: U
Kai	R	$a, 0$	c, d
	S	$b, 3$	$2, 4$

where a, b, c and d are some real numbers.

(a) [2] For what values of a, b, c and d is (R, T) a Nash equilibrium in this game?

Solution: No profitable deviation for Kai requires $a \geq b$. No profitable deviation for Lina requires $0 \geq d$.
Marks: [1] for each condition, penalize [0.5] each if use strict inequality.

(b) [2] For what values of a, b, c and d is playing S a dominant strategy for Kai?

Solution: If $b \geq a$ and $2 \geq c$.
Marks: [1] for each condition, don't penalize if use strict inequality.

(c) [2] For what values of a, b, c and d is playing U a *dominated* strategy for Lina?

Solution: There are no such values. Since $3 < 4$, U is a best response for Lina when Kai plays S, so U cannot be dominated strategy.
Marks: [1] for saying no such values, [1] for saying why.

(d) [8] Assume $a = 0$, $b = 3$, $c = 5$ and $d = 1$. Draw Kai's and Lina's best response functions and find all Nash equilibria in this game.

Solution: Denote Kai's strategy by $(p, 1-p)$ and Lina's strategy with $(q, 1-q)$.
Kai's best response is to play R or $(p = 1)$ if

$$5(1-q) \geq 3q + 2(1-q) \Rightarrow q \leq \frac{1}{2}$$

So his best response function is

$$p = BR_K(q) = \begin{cases} 1 & \text{if } q < \frac{1}{2} \\ [0,1] & \text{if } q = \frac{1}{2} \\ 0 & \text{if } q > \frac{1}{2} \end{cases}$$

Lina's best response is to play T or $(q = 1)$ if

$$3(1-p) \geq p + 4(1-p) \Rightarrow 3 \leq 4$$

Since $3 \leq 4$ is never true, T is never a best response for Lina. This is because playing U is a (strictly) dominant strategy. Thus, Lina's best response function is

$$q = BR_L(p) = 0 \quad \text{for all } p$$

The BR mapping must have p on one axis and q on one axis, and depict both best response functions (Lina's is a line, Kai's has two kinks).

Marks distribution

- Give [2] for calculating Kai's "critical" point $q = 1/2$, and noticing that Lina has a dominant strategy ("line" best response function).
- Give [4] for *drawing* the correct best-response functions. The axes and functions must be fully labelled for full marks. There is no need to write down the BR functions, but if no drawing is made, [1] can be given for writing down the BR functions.
- Give [2] for stating the NEs.

(e) [7] Now suppose Kai and Lina play a different game. This game is as follows: First, Kai chooses between the two-player, two-action simultaneous move games A and B. Then, Kai and Lina play either Game A or Game B depending on Kai's choice.

Game A can be represented in the following payoff matrix

		Lina	
		T	U
Kai	R	$3,2$	$0,1$
	S	$2,0$	$0,0$

Game B can be represented in the following payoff matrix

		Lina	
		Y	Z
Kai	W	$2,4$	$1,3$
	X	$1,2$	$0,1$

Find all subgame perfect equilibria (SPE) of this game.

Solution: Start from the second stage of the game and use backwards induction.

- If Game A is played, there are exactly two NE in this part: (R, T) and (S, U), giving Kai payoffs 3 and 0, respectively.
- If Game B is played, there is are exactly one NE in this part: (W, Y), giving Kai payoff 2.

If (R, T) is the eqm of Game A, then Kai chooses Game A (since $3 > 2$). If (S, U) is the eqm of Game A, then he chooses Game B (since $2 > 0$).
Hence, there are two SPE:

$$(A, ((R, T), (W, Y))) \quad \text{and} \quad (B, ((S, U), (W, Y)))$$

It may be easier to think about the equilibrium as "no regret at any point where I had to move".
Note: There is no reason to be super strict exactly how these strategy profiles are written down, but the reasoning must be correct.

Marks distribution

- Give [1] each for each NE in games A and B, for a total of [3]
- Give [2] for the first SPE that is found, and [3] for the second
  - i.e., if only one SPE is given, the maximum mark is [5]

6. Tanya owns a small plot of land. The only use of Tanya's plot of land is to grow rice. If a worker exerts effort x working on Tanya's land, then the amount of rice produced in one year (in tons) is given by

$$y = f(x) = \begin{cases} \ln x & \text{if } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Since the land plot is very small, it is not possible for more than one person to work on it. The market for rice is competitive, with a market price of 40 dollars per ton. Since Tanya cannot work on her land herself, she is considering asking Ivan to work for her. As Tanya's plot is in a very isolated area, Ivan is the only worker that could work on the plot. Ivan's cost of exerting effort x is given by

$$c(x) = 5x^2, \qquad x \geq 0$$

Ivan seeks to maximize his utility $U(x) = M - c(x)$, where M is his money income (in dollars). If Ivan does not work for Tanya, he can earn k dollars per year in an effortless call centre job. The market for call centre employees is competitive. If Ivan works on Tanya's plot, he cannot take the call centre job.

(a) [4] What is the efficient level of effort, x^E, for Ivan to exert working on Tanya's land plot?

Solution: The total surplus of Ivan working on Tanya's land is given by

$$TS(x) = 40f(x) - c(x) - k$$

If Ivan works $x \geq 1$ units on Tanya's land, then the efficient level of x is given by

$$x^* = \arg\max_x \{40 \ln x - 5x^2 - k\} = 2$$

It is efficient for Ivan to exert effort at all only if

$$40 \ln x^* - 5(x^*)^2 - k = 40 \ln 2 - 5(2)^2 - k \geq 0 \quad \Leftrightarrow \quad k \leq 40 \ln 2 - 20 \ (\approx 7.7259)$$

Hence, the efficient effort level is

$$x^E = \begin{cases} 2 & \text{if } k \leq 40 \ln 2 - 20 \ (\approx 7.7259) \\ 0 & \text{otherwise} \quad \Leftarrow \text{ should do call centre job instead} \end{cases}$$

Marks distribution

- Give [2] for interior solution (irrespective of whether k is included in TS function or not)
- Give [2] for arguing that zero will be optimal if k is above the cut-off value

(b) [8] Tanya wants to maximize the profit earned from her land plot. She cannot force Ivan to work for her or force him to exert any particular level of effort. However, she may hire Ivan as a worker and pay him a wage $w(y)$ that depends on the amount of rice he produces. Assume $k = 5$.

(i) If Tanya wants Ivan to exert effort x^E from (a), write down the incentive compatibility constraint (ICC) and participation constraint (PC) in her profit maximization problem.

(ii) Suggest a wage schedule $w(y)$ that is profit maximizing for Tanya. Explain carefully why your proposed wage schedule is profit maximizing. Calculate Tanya's profit.

Solution: Note that $x^E = 2$.

(i) The ICC is

$$\begin{aligned} & w(\ln(x^E)) - 5(x^E)^2 \geq w(\ln(x)) - 5x^2 \quad \text{for all } x \\ \text{or,} \quad & w(\ln(2)) - 20 \geq w(\ln(x)) - 5x^2 \quad \text{for all } x \end{aligned}$$

The PC is

$$\begin{aligned} & w(\ln(x^E)) - 5(x^E)^2 \geq 5 \\ \text{or,} \quad & w(\ln(2)) - 20 \geq 5 \end{aligned}$$

(ii) There is an infinite number of possible wage schedules. Here, we consider some linear wage schedule $w(y) = ay + b$ for some numbers a and b. Since $w(y)$ is differentiable in y (for $y > 0$), we can write the ICC as the FOC to the problem

$$\max_x \{(a \ln x + b) - 5x^2 - 5\} \quad \Rightarrow \quad \text{FOC:} \quad \frac{a}{x} - 10x = 0$$

Now, for incentive compatibility,

$$\frac{a}{x^E} - 10x^E = \frac{a}{2} - 20 = 0 \quad \Rightarrow a = 40$$

If Tanya wants to maximize her profit, she should leave no surplus for Ivan, i.e., the PC holds with equality. So

$$w(\ln(2)) - 20 = (40\ln(2) + b) - 20 = 5 \quad \Rightarrow b = 25 - 40\ln 2 (\approx -2.7259)$$

That is, the wage schedule is $w(y) = 40y + 25 - 40\ln 2$.
Another wage schedule that would accomplish the same thing is

$$w(y) = \begin{cases} 25 & \text{if } y = \ln 2 \\ 0 & \text{otherwise} \end{cases}$$

If student chooses this schedule, s/he still needs to show carefully why it satisfies IC and binds PC.
These wage schedules must be profit maximizing because (i) Ivan will choose the efficient level of effort, maximizing total surplus of the relationship, and (ii) Tanya can extract all total surplus of the relationship. Hence, there is no way she could be better off under some alternative wage schedule.
Tanya's profit is $40\ln 2 - 25 \approx 2.7259$

Marks distribution: [3] for part (i), [5] for part (ii)
Part (i)

- Constraints can be stated either in terms of x^E or directly in terms of 2
- Give [2] for ICC. If already assumed $w(.)$ is differentiable and stated the FOC rather than the general form, give [1].
- Give [1] for PC. Don't penalize if write with equality.

Part (ii)

- Give [1] for stating a wage schedule
- Give [1] for showing that the chosen wage schedule satisfies ICC
- Give [1] for showing that it binds PC
  - if doing the linear wage schedule solving for constants, these conditions hold by construction, so no need for any additional reasoning

- Give a total of [2] for arguing that the proposed wage schedule must be profit maximizing. This is because the efficient effort is induced so that total surplus is maximized [1], and because Tanya gets all surplus (worker gets no extra surplus) [1].

(c) [5] Keep the assumptions from (b). Ivan suggests that he and Tanya should share the revenue from the rice sales instead, with a share β for himself and a share $1-\beta$ for Tanya (with $0 \leq \beta \leq 1$). Show that this wage schedule cannot maximize Tanya's profit.

Solution: Revenues are $40y$, so this schedule is $w(y) = 40\beta y$. Ivan now solves

$$\max_x \{(40\beta \ln x) - 5x^2 - 5\} \quad \Rightarrow \quad \text{FOC:} \quad \frac{40\beta}{x} - 10x = 0$$

It is easy to see that the x chosen by Ivan is $x = 2\sqrt{\beta}$, meaning that the efficient effort level $x^E = 2$ will be chosen only if $\beta = 1$. But if $\beta = 1$, then Tanya gets nothing so her profit are clearly below the previous one. If $\beta < 1$, then Ivan's effort level is inefficiently low, so the total surplus must be below that in the previous question. This means that even if Tanya could extract all surplus, her profits would be lower than before. Hence, her profits must be lower so it cannot maximize Tanya's profit.

Marks distribution

- Give [1] for noting that Ivan gets $40\beta y$, or that Tanya gets $40(1-\beta)y$ (explicitly or implicitly)
- Give [2] for showing that Ivan will choose efficient level of effort only if $\beta = 1$
- Give a total of [2] for arguing that the scheme cannot be profit maximizing: this is because either the efficient effort is not chosen [1], or Tanya fails to extract some of the surplus [1]
  - other correct and clearly written arguments can also get full marks

7. Ashley and Mary-Kate are two utility maximizers who like to eat strawberries and chocolates. Ashley's utility from eating s strawberries and c chocolates is given by

$$u_A(s, c) = s + \ln c$$

Mary-Kate's utility from eating s strawberries and c chocolates is given by

$$u_M(s, c) = 3s + 2c$$

Whenever they have access to strawberries and chocolates, Ashley and Mary-Kate may trade these with each other but not with anyone else. Since they have access to a sharp knife, they can trade chocolate and strawberry pieces of any size (i.e., there is no need to trade only *whole* strawberries and chocolates).

(a) [3] Suppose the total endowment in this pure exchange economy is 5 strawberries and 3 chocolates. Find the set of Pareto efficient allocations (i.e., the Pareto set).

Solution: For interior solutions, we set $MRS_A = MRS_B$, so we have

$$c_A = \frac{3}{2}$$

That is, for the interior solutions, the chocolates are shared equally while the strawberries can be shared in any fashion.
In addition, there are some corner solutions (these are easiest to show in an Edgeworth box). They are first those allocations where Ashley consumes no strawberries and between 0 and $\frac{3}{2}$ chocolates, and second, where Mary-Kate consumes no strawberries and between 0 and $\frac{3}{2}$ chocolates.

Marks distribution

- The Pareto set can be stated either in words or algebra, or clearly marked in an Edgeworth box
- Give [1] for the interior set of PE allocations
- Give [1] each for the two segments of corner solutions, for a total of [2]

(b) [3] Now suppose that Ashley initially has all of the 5 strawberries while Mary-Kate has all of the 3 chocolates. Normalize the price of chocolates to 1. In the competitive equilibrium of this pure exchange economy, what is the price of strawberries? How many strawberries does Mary-Kate eat?

Solution: We normalize $p_c = 1$ so that p_s is the number of strawberries traded for each chocolate. There are two main ways of solving the problem:
Diagram: If the diagram has been drawn correctly, it will be obvious that we only need to care about the interior solutions and that the BC must coincide with Mary-Kate's utility function. Hence, the price ratio must be the same as the negative of Mary-Kate's MRS, or,

$$\frac{p_s}{p_c} = \frac{3}{2}$$

which gives $p_s = 3/2$ if we normalize $p_c = 1$. Putting c on y-axis and s on x-axis (can do reverse), we see that $(s_A, 3/2)$ must be on the line with slope $-3/2$ through $(5, 0)$, giving $s_A = 4$ so that Mary-Kate eats one strawberry.
Using market clearing conditions : As long as Ashley eats some strawberries, her demand for chocolates is given by $c_A(p_s) = p_s$ and she spends the rest of her income buying strawberries. For Mary-Kate, eating one chocolate always gives the same utility as 3/2 strawberries, so she buys only chocolates if $p_s > 2/3$ and only strawberries if $p_s < 2/3$. If $p_s = 2/3$, she's indifferent. From here, we can argue that the only price where the market can clear (on the interior) is such that $p_s = 2/3$.
Note that Mary-Kate's total wealth is 3, and since she spends 3/2 buying 3/2 chocolates, she has 3/2 left to spend on strawberries. Since they cost 2/3, she gets exactly one strawberry.

Marks distribution

- Give [3] if both price and number of strawberries provided is correct, as long as there is some evidence of any type of reasoning (it does not matter if it is super messy / sort of incomprehensible).

Partial marks

- Give [1] for realizing that the budget line must coincide with Mary-Kate's utility function, [1] for one strawberry, [1] for the correct price.

Note that if the Edgeworth box is drawn sufficiently neatly, it is possible to just read the quantity off the diagram. Don't penalize if this is the "method" used. The diagram in (a) may be used.

(c) [4] Now suppose that the total endowment (5 strawberries and 3 chocolates) originally belonged to Ashley and Mary-Kate's mother. The mother prefers that Mary-Kate eats 3 strawberries. However, since the girls' babysitter already gave 2.5 strawberries to Ashley and only 2.5 to Mary-Kate, the mother can only decide how to distribute the chocolates. How should the mother distribute the 3 chocolates to make sure that Mary-Kate eats 3 strawberries in the competitive equilibrium?

Solution: Again, it's easiest to do this in a graph. The solution will be on the interior. The only possible eqm allocation where Mary-Kate eats 3 strawberries is $(3, \frac{3}{2})$ for Mary-Kate and $(2, \frac{3}{2})$ for Ashley.

We now to find an initial endowment of chocolates e for Ashley so that a budget a line with slope $-3/2$ goes through $(2, \frac{3}{2})$ and $(2.5, e)$ (read from Ashley's origin). This can hold only for $e = 0.75$, i.e., the mother should give Ashley 0.75 chocolates, while Mary-Kate gets 2.25 chocolates.

Marks distribution

- Give [1] for writing down / labelling the only possible eqm allocation
- Give [2] for finding the number of chocolates to be given to one person, and [1] for the number of chocolates to be given to the second person
  - This can be done graphically if the graph is sufficiently neat. Don't penalize for this.
  - If not actually solve for the endowment, can still get [1] mark for saying that it's the line with slope ... that goes through ... and ...

8. Wen lives alone on an otherwise deserted tropical island. His time endowment is twenty-four hours per day. Every morning, he can effortlessly pick ten oysters on the beach. If he wants to eat more than ten oysters, then he needs to spend some time searching for them. If he spends h hours searching for oysters, then he will find h^2 oysters, but this is only true for the first four hours of searching. If he searches for oysters for more than four hours in a day, then he can find 3 oysters per hour for the remaining hours. In the tropical heat, oysters spoil quickly and cannot be saved for the next day. In other words, they must be eaten on the day that they are found.

   Wen's utility from eating y oysters and enjoying ℓ hours of leisure is given by

$$u(y, \ell) = \ln(y - \bar{y}) + \ln(\ell - \bar{\ell})$$

where $\bar{y} \geq 0$ and $\bar{\ell} \geq 0$ are some real numbers. Note that for Wen's utility function to be well-defined, he needs to eat $y > \bar{y}$ oysters and enjoy $\ell > \bar{\ell}$ hours of leisure. Wen is a utility maximizer.

(a) [1] What is the economic interpretation of $\bar{y}$ and $\bar{\ell}$?

Solution: They can be interpreted as minimum survival quantities. If Wen chooses only $y = \bar{y}$, then his utility is negative infinity. However, if he chooses $y > \bar{y}$, then his utility is a finite number. Same for ℓ.
Marks: [1] for any vaguely economic explanation; [0] for a purely mathematical explanation

(b) [4] In a diagram, draw Wen's production possibilities set (that is, the set of feasible combinations of oysters and leisure). Label the diagram clearly.

Solution: Here, I put h on the horizontal axis and y on the vertical axis (as in the book). Alternatively, we may put ℓ on the horizontal axis, in which case the image is "mirrorred". First note that if Wen doesn't work, he still gets 10 oysters, so (0,10) is on the PPF. If Wen works 4 hours, he gets $10 + 4^2 = 26$ oysters, so (4,26) is also on the PPF. The function connecting these points is convex (piece of quadratic parabola). Finally, if Wen works 24 hours, then he gets $10 + 4^2 + 3(20) = 86$ oysters, so (24,86) is also on the PPF. A line with slope 3 connects this point to (4,26). The production possibilities set is the area below this line, in the first quadrant, and to the left of 24.

Marks distribution

- Give [1] for marking (0,10).
- Give a total of [1] for the piece between (0,10) and (4,26): [0.5] for shape (convex), [0.5] for the fact that it connects to (4,26).
- Give a total of [1] for the piece between (4,26) and (24,86): [0.5] for shape (straight line), [0.5] for the fact that it connects to (24,86).
- Give [1] for marking the *set* of feasible allocations, e.g., by shading or verbal description.

(c) [5] Suppose $\bar{y} = 20$ and $\bar{\ell} = 6$. How many oysters will Wen eat in a day?

Solution: The solution may be either on the linear or the convex part. However, if you draw a utility curve of approximately the right shape (it's a regular, well-behaved CD but asymptotic to $y = 20$ and $h = 18$) in the diagram, it is obvious that the solution must be on the linear part. The equation for the PPF on the linear part, i.e., on the line through (4,26) and (24,86), can be written $y = 3h + 14$. Plugging the time constraint $h + \ell = 24$ into the utility function, we can write

$$u(y, h) = \ln(y - 20) + \ln(18 - h)$$

.

The easiest way to solve this problem is to solve

$$\max_{h,y}\{u(y,h) \text{ s.t. } y = 3h + 14\}$$

and verify that $h \geq 4$. Plugging in the constraint, we have

$$\max_{h}\{\ln(3h-6) + \ln(18-h)\} \quad \Leftrightarrow \quad \max_{h}\{\ln 3 + \ln(h-2) + \ln(18-h)\}$$

with FOC

$$\frac{1}{h-2} - \frac{1}{18-h} = 0 \quad \Rightarrow \quad h = 10$$

Since Wen works 10 hours, he eats $10 + 4^2 + 3(6) = 16 + 18 = 44$ oysters. Alternatively, we can use the tangency condition MRS = MRTS directly. This gives

$$-\frac{\frac{1}{18-h}}{\frac{1}{y-20}} = -\frac{y-20}{18-h} = -\frac{1}{3}$$

Now, plugging in $y = 3h + 14$, we can solve for h

$$\frac{(3h+14)-20}{18-h} = \frac{1}{3} \quad \Rightarrow \quad h = 10$$

Marks distribution

- Give [2] for arguing that solution must be on the linear part (either graphically, or otherwise)
- For a total of [2], give [1] for writing down the maximization problem and [1] for the correct technology constraint **OR** [1] for the tangency condition and [1] for the correct technology constraint
- Give [1] for solving for $y = 44$
  - if the PPF (constraint in the utility maximization problem) is not drawn correctly in (b) and this affects the calculations in this question, a maximum of [3] may still be given
  - if the mistake in (b) significantly simplifies the problem (e.g., PPF is drawn with no kinks), a maximum of [2] may be given
  - if write $y = 3h + m$ where $m \neq 14$ but the PPF is drawn correctly in (b), a maximum of [4] may be given

End of examination
Total pages: 16
Total marks: 100