MAT 2379, Introduction to Biostatistics

Chapter 13. Paired Samples

A typical example of *dependent* data sets are repeated observations that are taken on the same individuals.

Example: the weight x before a weight loss program compared the weight y after the program, the midterm grade x compared with the final exam grade y, the blood pressure x before a physical exercise compared with the blood pressure y after the exercise, etc.

In this case the two samples have the same size. The two sample are also called "paired samples" because they always arise in pairs.

We denote with $X_1, \ldots, X_n$ the observations from a population with mean μ_X and with $Y_1, \ldots, Y_n$ the observations from a population with mean μ_Y. The two data sets are supposed to be dependent. We would like to compare μ_X with μ_Y. In other words, we want to decide whether the difference $\mu_D = \mu_X - \mu_Y$ is positive ore negative.

For this we calculate the differences $D_1 = X_1 - Y_1$, $D_2 = X_2 - Y_2$, etc. These differences will constitute a new data set that will be used for drawing statistical conclusions. The sample mean $\bar{D}$ and the sample standard deviation S_d calculated from the "difference" data set will play an important role. We will assume that these differences are normally distributed. (Hence, the sample size can be small.)

13.1 Interval estimation for μ_D

The confidence interval for μ_D is similar to that constructed in Chapter 10:

$$\bar{d} \pm t \left(\frac{s_d}{\sqrt{n}} \right)$$

where t corresponds to the T_{n-1} distribution and the desired level of confidence.

Example 1. (p.347 of "Statistics for the Life Sciences" by Samuels and Witmer) The compound m-chloro-phenyl-piperazine (mCPP) is thought to affect appetite and food intake. 9 women took mCPP for two weeks (session 1), then took nothing for two weeks, and then took a placebo for two weeks (session 2). The weight loss (in kg) for each women was recorded after each of the two sessions. Here is the table of the data set: (column 1-mCPP, column 2-placebo, column 3-difference)

Session 1 (x)	Session 2 (y)	Difference $d = x - y$
1.1	0.0	1.1
1.3	-0.3	1.6
1.0	0.6	0.4
1.7	0.3	1.4
1.4	-0.7	2.1
0.1	-0.2	0.3
0.5	0.6	-0.1
1.6	0.9	0.7
-0.5	0.2	-0.7

Find a 95% confidence interval for the average difference μ_D. Interpret the result.

From the data set we can calculate the mean and the standard deviation of the differences: $\bar{d} = 0.76$ and $s_d = 0.88$. A 95% confidence interval for μ_D is

$$0.76 \pm 2.306 \left(\frac{0.88}{\sqrt{9}} \right), \quad \text{or} \quad 0.76 \pm 0.68, \quad \text{or} \quad [0.08;\ 1.44]$$

Since the interval contains only positive values, we are 95% confident that μ_X (which corresponds to mCPPP) is greater than than μ_Y (which corresponds to placebo). More precisely, we can say that the population average weight loss (in a two week period) is between 0.08 kg and 1.44 kg greater when taking mCPP than when taking a placebo.

13.2 Hypothesis testing for μ_D

The test statistic to be used is

$$T_0 = \frac{\bar{D} - 0}{S_d/\sqrt{n}}$$

which has an approximate T_{n-1} distribution. Here are the three cases:

$$\textbf{Case I}: \begin{cases} H_0 : \mu_D = 0 \\ H_1 : \mu_D > 0 \end{cases} \qquad p - \text{value} = P\left(T \geq \frac{\bar{d} - 0}{s_d/\sqrt{n}} \right)$$

$$\textbf{Case II}: \begin{cases} H_0 : \mu_D = 0 \\ H_1 : \mu_D < 0 \end{cases} \qquad p - \text{value} = P\left(T \leq \frac{\bar{d} - 0}{s_d/\sqrt{n}} \right)$$

$$\textbf{Case III}: \begin{cases} H_0 : \mu_D = 0 \\ H_1 : \mu_D \neq 0 \end{cases} \qquad p - \text{value} = \begin{cases} 2P\left(T \geq \frac{\bar{d}-0}{s_d/\sqrt{n}} \right) & \text{if } \bar{d} > 0 \\ 2P\left(T \leq \frac{\bar{d}-0}{s_d/\sqrt{n}} \right) & \text{if } \bar{d} < 0 \end{cases}$$

Example 2. (p. 319 in "Statistical Methods in the Biological and Health Sciences" by Susan Milton) A study is conducted to determine the effect of a home meter for helping diabetics control their blood glucose level. A random sample of 30 diabetics participate in this study. Blood glucose levels were obtained for each patient before they were taught how to use the meter, and again after they had utilized the meter for several weeks. A mean sample difference of 2.78mmol/liter with a sample standard deviation of 6.05 mmol/liter was recorded (subtraction done in the order of "before" minus "after"). Is there sufficient evidence to claim that the monitor is effective in helping patients to reduce their blood glucose levels? Support your conclusion using a test of hypotheses. Use the level $\alpha = 0.01$.

Let μ_X be the average blood glucose level for patients who do not know how to use the meter and μ_Y be the average blood glucose level for patients who have used the meter for several weeks. We would like to test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X > \mu_Y$. The observed value of the test statistic is:

$$t_0 = \frac{2.78 - 0}{6.05/\sqrt{30}} = 2.52$$

p-value$= P(T > 2.52)$ where T has a T distribution with 29 d.f. From Table 17.4, we see that the p-value is between 0.005 and 0.01. Since the p-value is smaller than 0.01, we reject H_0 and conclude that the meter is effective in reducing the blood glucose level.