WILFRID LAURIER UNIVERSITY
ECONOMICS 325 - GAME THEORY AND ECONOMIC ANALYSIS
ASSIGNMENT # 6 - ANSWERS

Q1 (14 marks)
Competition in Quantities (Homogeneous Products)

Suppose that the inverse market demand for a commodity is given by

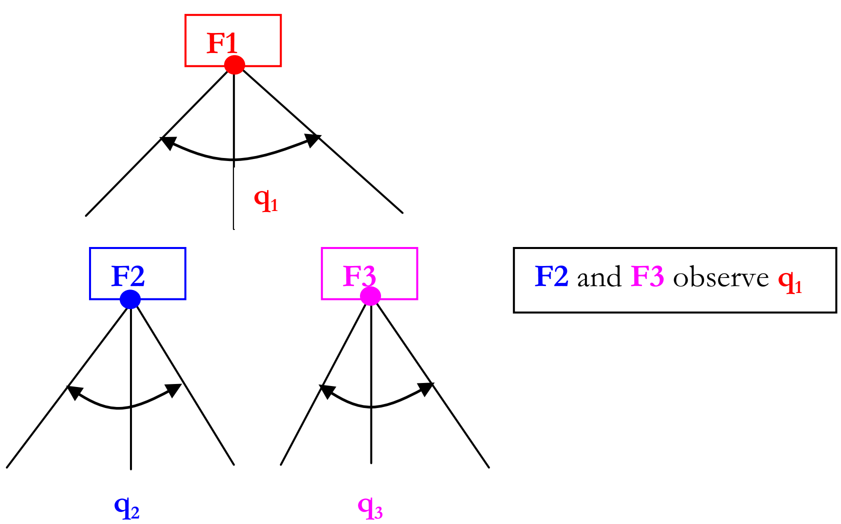
$$P = 240 - \frac{1}{3}Q$$

The cost curves of the three firms which could serve this market are

$$TC_1(q_1) = 30q_1 \quad \text{and} \quad TC_2(q_2) = \frac{1}{3}q_2^2 \quad \text{and} \quad TC_3(q_3) = \frac{1}{3}q_3^2$$

Suppose there are three firms serving the market. Firm 1 moves first by choosing its output level. In the second stage, Firms 2 and 3 observe Firm 1's output level and move simultaneously. Draw the game tree. Find the subgame-perfect equilibrium quantities of this game. Determine the resulting profits for the three firms. Compare the outcome of this game to the outcome obtained in **Assignment 5, Q3 part (c)**. What is the impact of adding a firm to the second stage?

5 marks in total for the drawing the game tree correctly
1/2 for showing F1 in the first node
1/2 for showing that F1 picks q1 out of an infinite number of possibilities
1 for showing F2 and F3 as making simultaneous decisions
1 for saying that F2 and F3 observe F1's output q1
1/2 each for showing the F2 and F3 chose their output from an infinite number of possibilities



You canNOT put F2 and F3 together as they are not making joint decisions.

Start at the last (second) stage there is a **simultaneous move game** between two identical firms in a market where the products are prefect substitutes. **F2** and **F3** take the output of **F1** as given

$$P = 240 - \frac{1}{3}Q = 240 - \frac{1}{3}(q_1 + q_2 + q_3)$$

1 $\pi_2(q_1,q_2,q_3) = [P(q_1,q_2,q_3) - AC_2] \times q_2 = [240 - \frac{1}{3}q_1 - \frac{1}{3}q_2 - \frac{1}{3}q_3 - \frac{1}{3}q_2] \times q_2$

$\pi_2(q_1,q_2,q_3) = 240q_2 - \frac{1}{3}q_1q_2 - \frac{2}{3}(q_2)^2 - \frac{1}{3}q_3q_2$

Profit maximizing condition (F.O.C.)

1 $\frac{d\pi_2}{q_2} | q_1, q_3 = 240 - \frac{1}{3}q_1 - \frac{4}{3}q_2 - \frac{1}{3}q_3 = 0$

Alternatively,

1/4 $TR_2 = P(q_1,q_2,q_3) \times q_2 = \left[240 - \frac{1}{3}(q_1 + q_2 + q_3)\right] \times q_2 = 240q_{21} - \frac{1}{3}q_1q_2 - \frac{1}{3}(q_2)^2 - \frac{1}{3}q_2q_3$

1/4 $\Rightarrow \quad MR_2 | q_1, q_3 = \frac{dTR_2}{dq_2} | q_1, q_3 = 240 - \frac{1}{3}q_1 - \frac{2}{3}q_2 - \frac{1}{3}q_3$

1/2 $TC_2(q_2) = \frac{1}{3}q_2^2 \quad \Rightarrow \quad MC_2 = \frac{dTC_2(q_2)}{dq_2} = \frac{2}{3}q_2$

Profit maximizing condition (F.O.C.)

$MR_2(q_1,q_2,q_3) | q_1, q_3 = MC_2(q_2)$

$240 - \frac{1}{3}q_1 - \frac{2}{3}q_2 - \frac{1}{3}q_3 = \frac{2}{3}q_2$

1 $\frac{4}{3}q_2 = 240 - \frac{1}{3}q_1 - \frac{1}{3}q_2$

$q_2 = 240 \times \frac{3}{4} - \frac{1}{3} \times \frac{3}{4}q_1 - \frac{1}{3} \times \frac{3}{4}q_2$

1 $\Rightarrow \quad CBR2: \quad q_2 = 180 - \frac{1}{4}q_1 - \frac{1}{4}q_3$ **(1)**

1/2 **F2** and **F3** are identical; their products are perfect substitutes and face same quantity chosen by **F1.**

1 $\Rightarrow \quad CBR3: \quad q_3 = 180 - \frac{1}{4}q_1 - \frac{1}{4}q_2$ **(2)**

1/2 The **BR of F2** to the **BR of F3** when taking F1's output q_1 as given, that is solving for $q_2(q_1)$ and $q_3(q_1)$, [sub (2) into (1)]

$q_2 = 180 - \frac{1}{4}q_1 - \frac{1}{4}q_3$

$q_2 = 180 - \frac{1}{4}q_1 - \frac{1}{4}\left[180 - \frac{1}{4}q_1 - \frac{1}{4}q_2\right]$

$q_2 = 180 - \frac{1}{4}q_1 - \frac{1}{4}180 + \frac{1}{4} \times \frac{1}{4}q_1 + \frac{1}{4} \times \frac{1}{4}q_2$

$q_2 = 180 \times \left[1 - \frac{1}{4}\right] - \frac{1}{4} \times \left[1 - \frac{1}{4}\right]q_1 + \frac{1}{16}q_2$

$$\left[1-\frac{1}{16}\right]q_2 = 180\times\frac{3}{4}-\frac{1}{4}\times\frac{3}{4}q_1$$

1/2 $$\left[\frac{15}{16}\right]q_2 = 180\times\frac{3}{4}-\frac{3}{16}q_1$$

$$q_2 = 180\times\frac{3}{4}\times\frac{16}{15}-\frac{3}{16}\times\frac{16}{15}q_1$$

1 $\Rightarrow$ CBR2: $q_2 = 144-\frac{1}{5}q_1$

1/2 Since **F2** and **F3** have the same cost function, are producing identical products and move simultaneously

1/2 $\Rightarrow$ CBR3: $q_3 = 144-\frac{1}{5}q_1$

Alternatively (3 marks),

1/2 since the products are perfect substitutes, the firms have identical cost structures and they make simultaneous choices, in equilibrium and only in equilibrium these firms will produce the same quantity. So that

1/2 $q_2{}^* = q_3{}^* = q^*$ **(3)**

To find the equilibrium quantity of **F2** and **F3** then sub **(3)** into either (1) or (2)

$$q_2 = 180-\frac{1}{4}q_1-\frac{1}{4}q_3$$

$$q^* = 180-\frac{1}{4}q_1-\frac{1}{4}q^*$$

1/2

$$\left[1+\frac{1}{4}\right]q^* = 180-\frac{1}{4}q_1$$

$$\left[\frac{5}{4}\right]q^* = 180-\frac{1}{4}q_1$$

$$q^* = 180\times\frac{4}{5}-\frac{1}{4}\times\frac{4}{5}q_1$$

$$q^* = 144-\frac{1}{5}q_1$$

1/2 $\Rightarrow$ $q_2^C = 144-\frac{1}{5}q_1$ **(4)**

1/2 Since **F2** and **F3** have identical cost structures and produce identical goods then

1/2 $\Rightarrow$ $q_3^C = 144-\frac{1}{5}q_1$ **(5)**

Now move back to **F1**

Note that $TC_1(q_1) = 30q_1 \quad \Rightarrow AC_1(q_1) = \frac{TC_1(q_1)}{q_1} = 30$

Profit function of F1 is

$$\pi_1(q_1,q_2,q_3) = [P(q_1,q_2,q_3) - AC_1] \times q_1$$
$$= \left[240 - \frac{1}{3}(q_1+q_2+q_3) - 30\right] \times q_1$$
$$= 210q_1 - \frac{1}{3}(q_1)^2 - \frac{1}{3}q_2q_1 - \frac{1}{3}q_3q_1$$

Because this is a game of complete information, **F1** knows the **CBR2** and **CBR3**. Moreover, **F1** knows the solution to the simultaneous Cournot game that takes places after it choses its quantity q_1. **F1** will use this information to its advantage by taking into account that **F2** and **F3** will decrease their output if **F1** increases its output, that is, by taking into account that the quantities produced by **F2** and **F3** will be given by (4) and (5) and that these BR functions are negatively slopped.

1/2 **F1** takes BR functions of **F2** and **F3** by plugging **(4)** and **(5)** into profit its profit calculations.

$$\pi_1(q_1,q_2,q_3) = 210q_1 - \frac{1}{3}(q_1)^2 - \frac{1}{3}q_2q_1 - \frac{1}{3}q_3q_1$$
$$\pi_1(q_1) \mid CBRF2, CBRF3 = 210q_1 - \frac{1}{3}(q_1)^2 - \frac{1}{3}\left[144 - \frac{1}{5}q_1\right]q_1 - \frac{1}{3}\left[144 - \frac{1}{5}q_1\right]q_1$$
$$\pi_1(q_1) \mid CBRF2, CBRF3 = 210q_1 - \frac{1}{3}(q_1)^2 - \frac{2}{3}\left[144 - \frac{1}{5}q_1\right]q_1$$
$$= 210q_1 - \frac{1}{3}(q_1)^2 - \frac{2}{3}144q_1 + \frac{2}{15}(q_1)^2$$

1/2

$$= \left[210 - \frac{2}{3}144\right]q_1 - \left[\frac{1}{3} - \frac{2}{15}\right](q_1)^2$$
$$= (210-96)q_1 - \frac{5-2}{15}(q_1)^2$$

1/2 $$\pi_1(q_1) \mid CBR2, CBR3 = 114q_1 - \frac{3}{15}(q_1)^2$$

The profit max cond of **F1** is

1/2 $$\frac{d\pi_1(q_1)}{dq_1} \mid CBR2, CBR3 = 114 - \frac{6}{15}q_1 = 0$$

Consequently, **F1**, the **Stakelberg leader** produces

1/2 $$q_1^{SL} = 285$$

to maximize profits.

Now that we know how much **F1** produces we can use the **(7)** and **(8)** to find how much the followers, **F2** and **F3** will produce. Sub $q_1^{SL} = 285$ to get

1/2

$$q_2^C = 144 - \frac{1}{5}q_1^{SL}$$
$$= 144 - \frac{1}{5} \times 285 = 87$$

Since **F2** and **F3** have identical costs and produce identical products that consumers consider to be perfect substitutes then

1/2 $\Rightarrow \quad q_3^{SF} = q_2^{SF} = 87$

We are now ready to find price, costs and profits

$$P^S = 240 - \frac{1}{3}(q_1^{SL} + q_2^{SF} + q_3^{SF})$$

1/2 $$= 240 - \frac{1}{3}(285 + 87 + 87)$$

$$= 240 - \frac{1}{3} \times 459 = 87$$

1/2 $$\pi_1^{SL}(q_1^{SL}, q_2^{SF}, q_3^{SF}) = [P^S - AC_1] \times q_1^{SL} = [87 - 30] \times 285 = 16{,}245$$

Note: Stakelberg leader makes **smaller profits** when it is followed by two rather than by only one firm.

1/2 $$\pi_1^{SL}(A6Q1) = 16{,}245 < \pi_1^{SL}(A5Q3c) = 22{,}500$$

With **F2** and **F3** having identical costs, producing identical goods and producing the same quantity then **F2** and **F3** make the same profits. But need to calculate the ATC of **F2** and **F3**.

$$TC_i(q_i) = \frac{1}{3} q_i^2 \quad \text{for } i = 2,3$$

$$\Rightarrow \quad AC_i(q_i) = \frac{TC_i(q_i)}{q_i} = \frac{1}{3} q_i \quad \text{for } i = 2,3$$

So that when $q_i^{SF} = 87$ then

$$AC_i(q_i) = \frac{1}{3} q_i = \frac{1}{3} \times 87 = 29 \quad \text{for } i = 2,3$$

1/2 $$\pi_i^{SF}(q_1^{SL}, q_2^{SF}, q_3^{SF}) = [P^S - AC_i] \times q_i^{SF} = [87 - 29] \times 87 = 5{,}046 \ \text{ for } i = 2,3$$

1/2 $$\pi_2^{SF}(A6Q1) = 5{,}046 < \pi_2^{SF}(A5Q3c) = 7{,}350$$

So that both followers make **smaller profits** than if there was only one follower.

1/2 In conclusion, the market Q increases when adding one more follower, market P declines and profits of all firms fall.

Q2 (27 marks)

Sequential Equilibrium

You are considering a leveraged buy out of Corporation X. The stock of X is **worth** either \$0/share or \$5/share. The management of the company knows what it is worth, and if it puts the **company up for sale it asks \$2/share for 10 thousand shares outstanding**. All you know is that the **probability** that the company is worth \$5/share is **50%**. It costs management **\$50,000** to cook the books if it has to make the company look better that it really is.

1 **(a)** What is the company worth to the buyer under the good (V) and the bad (W) states of nature?

- Under the **good** state of Nature

1/2 $\quad V = \dfrac{\text{Stock Price(G)}}{\text{share}} \times \text{No. of shares} = \$5 \times 10{,}000 = \$50{,}000$

- Under the **bad** (W) states of nature

1/2 $\quad W = \dfrac{\text{Stock Price(B)}}{\text{share}} \times \text{No. of shares} = \$0 \times 10{,}000 = \$0$

3 **(b)** If there is a leverage buy out, at what price does the company trade hands? What are the profits to the **buyer** and to the **seller** under the **good** and the **bad** state of nature?

If the company trades, changes owner, the price of the company is

1 $\quad p = \dfrac{\text{Sale Price}}{\text{share}} \times \text{No. of shares} = \$2 \times 10{,}000 = 20{,}000$

Buyer's profits

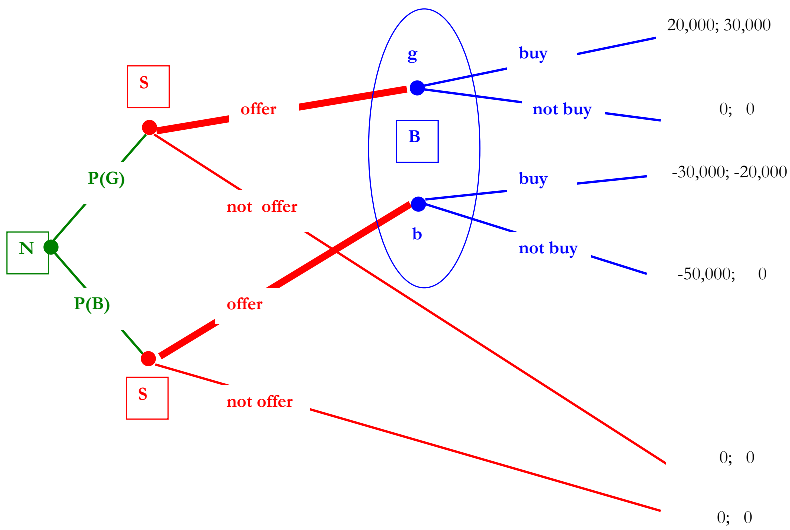
- Under the **good** state of Nature:
  1/2 $\quad V - p = V - p = \$50{,}000 - \$20{,}000 = \$30{,}000$
- Under the **bad** (W) states of Nature:
  1/2 $\quad W - p = \$0 - 20{,}000 = -\$20{,}000$

Seller's profits

- Under the **good** state of Nature
  1/2 $\quad p = \$20{,}000$
- Under the **bad** (W) states of nature, seller must cook books (remember company sells at a single price, so price is not informative here)
  1/2 $\quad p - c = \$20{,}000 - 50{,}000 = -30{,}000$

5 **(c)** Draw the game tree showing the payoffs first to the **seller** and then the **buyer**.

2 total at 1/6 each correct value x 16 values
2 total at 1/5 each correctly labelled branch of the tree x 14 labels
1/2 total at 1/20 each for correctly labelling the nodes x 10 labels
1/2 total at 1/4 each for circling the nodes of the buyer



17 **(d)** Using conditional probability arguments and Bayes' rule, determine the sequential equilibrium of the game. **Prove** that the proposed equilibrium is indeed a sequential equilibrium.

7 Proposed sequential Equilibrium

Note since $c = 50{,}000 > p = 20{,}000$ we have a separating equilibrium.
It is too expensive for the current owners to cook the books to sell the company.

1 Note that $p - c = 80{,}000 - 40{,}000 = 40{,}000 < q = \$50{,}000$

So there is a separating equilibrium

Seller's strategy

- 1 **Seller** with **good** (high value) company **offers company for sale**
- 1 **Seller** with **bad** (low value) company does **NOT** offer company for sale

Buyer's belief

- 1 $\text{prob}(\text{node g} \mid \text{offer}) = 1$
- 1 $\text{prob}(\text{node b} \mid \text{offer}) = 0$

Buyer's strategy

1 **Given beliefs**
1 **buyer's strategy:** buy whatever is offered for sale

10 Proof:
Start with buyer (last mover)

Buyer observes that for **seller**

- 1 If **seller** knows company is **good** (high) value **seller** wants to sell company since
  $V_S(\text{sale} \mid G) = 20{,}000 > V_S(\text{no sale}) = 0$
  $\Rightarrow$ **Seller** offers the company for sale with $\text{prob}(O \mid G) = 1$
- 1 If **seller** knows company is **bad** (low) value and must cook up costs of 50,000
  $V_S(\text{sale} \mid \text{bad}) = -30{,}000 < V_S(\text{no sale}) = 0$
  $\Rightarrow$ **seller** offers company for sale with $\text{prob}(O \mid B) = 0$

Buyer then knows that the probability of an offer (the **marginal probability** of an offer regardless of whether the company is in good or bad shape) is

1 $$p(O) = p(G)p(O \mid G) + p(B)\text{prob}(O \mid B) = p(G) \times 1 + p(B) \times 0 = p(G)$$

Buyer's belief (posterior) that the company is **good** given that it is offered for sale is

1 $$p(G \mid O) = \frac{p(G) \times p(O \mid G)}{p(O)} = \frac{p(G) \times 1}{p(G)} = 1$$

1/2 $\Rightarrow$ **buyer** believes in **node g** with probability 1
and **buyer believes** that in **node b** with probability

1/2 $$p(B \mid O) = \frac{p(B) \times p(O \mid B)}{p(O)} = \frac{p(B) \times 0}{p(G)} = 0.$$

Given **buyer's belief,** the **expected value to the buyer** of buying the company given

1/2 $EV_B(\text{buy}) = p(\text{node g} | O) \times [V - p] + p(\text{node b} | O) \times [W - p]$

1/2 $EV_B(\text{buy}) = 1 \times [V - p] + 0 \times [W - p] = V - p = 50{,}000 - 20{,}000 = 30{,}000$

1/2 $\Rightarrow \quad EV_B(\text{buy}) = 30{,}000 > EV_B(\text{no buy}) = 0$

1/2 $\Rightarrow$ Believing that only the company in the good state of nature is offered for sale, **buyer's BR** to buy the company when it is offered for sale

Move back to seller (second mover)

1/2 Seller takes into account **buyer's BR** function

BR of the Seller

- 1 with the **good** (high value) company, i.e., under the good state of nature, is to offer the company for sale since

  $V_S(\text{sale} | G) = 20{,}000 > V_S(\text{no sale}) = 0$

- 1 with **bad** (low value) company, i.e., under the bad state of nature, is **NOT** to offer the company for sale since

  $V_S(\text{sale} | \text{bad}) = -30{,}000 < V_S(\text{no sale}) = 0$

End of proof

1/2 Therefore we have a separating equilibrium and complete market success

1 **(d)** Given your answer in (d), if offered for sale should you buy the company at this price?

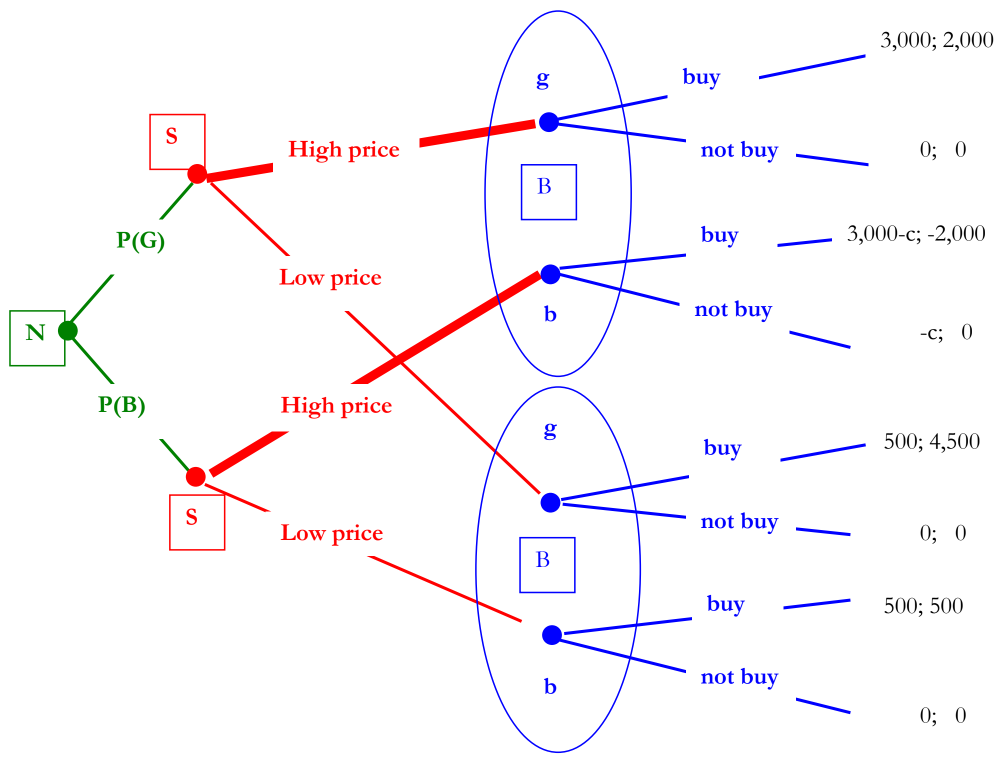
Buyer believes that the company offered for sale is of **good** (high) value and buys the company.

Q3 (59 marks)
Market for Lemons

Apply the following the values $V = \$5{,}000, W = \$1{,}000, p = \$3000, q = \$500$ to the Caveat Emptor Game. Ignore near market failure analysis and mixed strategy equilibrium case.

5 **(a)** Draw the game tree showing first the payoff to the seller and the payoff to the **buyer**.

2 total at 1/8 each correct value x 16 values
2 total at 1/7 each correctly labelled branch of the tree x 14 labels
1/2 total at 1/18 each for correctly labelling the nodes x 9 labels
1/2 total at 1/4 each for circling the nodes of the buyer



9 **(b)** Using conditional probability arguments and Bayes' rule, determine the **range of values** for $c > 0$ that give rise to complete and partial market success. Ignore near market failure.

3 Note $V - q = \$4{,}500 > V - p = 2{,}000 > W - q = \$500 > 0 > W - p = -\$4{,}500$

Possible values for c

- **2** $c > p = 3{,}000$
  It is not profitable for **seller** to clean up a **BAD item** and sell it at a high since the clean-up costs are too high.
- **2** $p - c = 3{,}000 - c \le q = 500 \quad \Rightarrow \quad 2{,}500 < c$
  Seller with **bad item** makes more money from NOT cleaning the item and selling it at low price rather than cleaning it and selling at high price
- **2** $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$
  Seller with **bad item** makes more money from cleaning item and selling at high price than not cleaning it and selling at low price

45 **(c)** For each range of c, identify the sequential equilibrium and prove that the candidate equilibrium is indeed a sequential equilibrium.

20 Case 1: $c > p = 3{,}000$
When item is bad, it is not profitable for seller to clean up the item to sell it at a high

7 **Proposed Sequential Equilibrium**
1 **Note that** $c > p = 3{,}000$
So there is a separating equilibrium

Seller

- **1** with **good** item offers item for sale at a **high price p**
- **1** with **bad** item offers item for sale at **low price q**

Buyer's belief

- **1** $\text{prob(node g | high price p)} = 1$
- **1** $\text{prob(node b | low price q)} = 1$

1 Given beliefs
1 buyer's strategy: buy whatever is offered for sale

13 Proof of Case 1: $c > p = 3{,}000$

Start with buyer (last mover)

Buyer observes that for **seller**

- **1** If **seller** knows item is **good** then offers to sell at **high price (p)** since
  $V_S(\text{sell at } p \mid G) = 3{,}000 > V_S(\text{sell at } q \mid G) = 500$
  $\Rightarrow$ Offer item for sale at p with $\text{prob}(p \mid G) = 1$

- **1** If **seller** knows item is bad and must incur fix up costs of c
  $V_S(\text{sale at } p \mid B) = 3{,}000 - c < 0 < V_S(\text{sale at } q \mid B) = 500$
  $\Rightarrow$ Offer item for sale at **low price** q with $\text{prob}(q \mid B) = 1$

Buyer then knows that

- **1** probability that **seller** offers item at a **high price p** is

$$\begin{aligned}\text{prob}(p) &= p(G)p(p \mid G) + p(B)p(p \mid B)\\ &= p(G)\times 1 + p(B)\times 0\\ &= p(G)\end{aligned}$$

Therefore, **buyer's belief** that item is **good** given that it is offered at a **high price p**

1 $$\text{prob}(G \mid p) = \frac{p(G)p(p \mid G)}{p(p)} = \frac{p(G)\times 1}{p(G)} = 1$$

1/2 $\Rightarrow$ **buyer** believes in **node g** with probability 1 if item offered at **high price p**

- **1** probability that **seller** offers item at a **low price q** is

$$\begin{aligned}\text{prob}(q) &= p(G)p(q \mid G) + p(B)p(q \mid B)\\ &= p(G)\times 0 + p(B)\times 1\\ &= p(B)\end{aligned}$$

Therefore, **buyer's belief** that item is **bad** given that it is offered at a **low price q**

1 $$\text{prob}(B \mid q) = \frac{p(B)p(q \mid B)}{p(q)} = \frac{p(B)\times 1}{p(B)} = 1$$

1/2 $\Rightarrow$ **buyer** believes in **node b** with probability 1 if item offered at **low price q**

Given **buyer's beliefs, buyer**

- **buys** item if offered at **high price p** since

1/2 $EV_B(\text{buy at p}) = p(G\,|\,p)\times 2{,}000 + p(B\,|\,p)\times[-2{,}000]$
$= 1\times 2{,}000 + 0\times[-2{,}000] = 2{,}000$

and $EV_B(\text{not buy at p}) = p(G\,|\,p)\times 0 + p(B\,|\,p)\times 0 = 0$

1/2 So that $EV_B(\text{buy at p}) = 2{,}000 > EV_B(\text{not buy at p}) = 0$

1 and the **buyer's BR** is to buy the item sold at a **high price p**

- **buys** item if offered at **low price q** since

1/2 $EV_B(\text{buy at q}\,|\,\text{node b}) = p(G\,|\,q)\times 4{,}500 + p(B\,|\,q)\times 500$
$= 0\times 4{,}500 + 1\times 500 = 500$

and $EV_B(\text{not buy at q}) = p(G\,|\,q)\times 0 + p(B\,|\,q)\times 0 = 0$

1/2 So that $EV_B(\text{buy at q}\,|\,\text{node b}) = 500 > EV_B(\text{not buy at q}) = 0$

1 and **buyer's BR** is to buy the item sold at **low price q**.

Move back to seller (second mover)

Given **buyer's BR function**

Seller

- 1 with **good** item offers item for sale at **high price p**
  $EV_S(\text{sell at p}\,|\,G) = 3{,}000 > EV_S(\text{sell at q}\,|\,G) = 500$
- 1 with **bad** item offers item for sale at **low price** q since it pays to incur clean up costs
  $EV_S(\text{sell at q}\,|\,B) = 500 > 0 > EV_S(\text{sell at p}\,|\,G) = 3000 - c$

End of proof of Case 1

$\Rightarrow$ **In Case 1** we have a separating equilibrium and complete market success

5 **Case 2:** $p - c = 3{,}000 - c \le q = 500 \quad \Rightarrow \quad 2{,}500 < c$

Seller with **bad item** makes more money from not cleaning item and selling it at low price rather than cleaning it and selling at high price

3 **Candidate for Sequential Equilibrium**

Seller

- 1/2 with **good** item offers item for sale at a **high price (p)**
- 1/2 with **bad** item offers item for sale at **low price (q)**

Buyer's belief

- 1/2 $\text{prob}(\text{node g}\,|\,\text{high price p}) = 1$
- 1/2 $\text{prob}(\text{node b}\,|\,\text{low price q}) = 1$

1 Given beliefs **buyer's strategy:** buy whatever is offered for sale

2 **Proof of Case 2:** $q = 500 > p - c = 3{,}000 - c \quad \Rightarrow \quad c > 2{,}500$

1/2 **Note**: in this case when item is bad, it is not profitable for seller to clean up the item to sell it at a high so this is basically the same as Case 1.

1/2 Therefore the proof would be **identical to Case 1 (as is not repeated here)**
1 $\Rightarrow$ In case 2 we have a separating equilibrium and complete market success

The next cases deal with $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$

Seller with bad item makes more money from cleaning item and selling at high price than not cleaning it and selling at low price.

In addition, we need to also take into account whether the **buyer will buy or not** the item, that is whether, $EV_B(\text{buy at } p) > 0$ or $EV_B(\text{buy at } p) < 0$.

This is sub-divided into two cases

- **Case 3:** $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$ **and** $EV_B(\text{buy at } p) > 0$
- **Case 4**: $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$ **and** $EV_B(\text{buy at } p) < 0$.
  Case 4 deals with mixed strategies and near market failure and was not part of Assignment # 6 and will not be tested on the final.

20 Case 3: $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$ and $EV_B(\text{buy at } p) > 0$

7 Candidate for Sequential Equilibrium

1 Note that $q = 500 < p - c = 3{,}000 - c \quad \Rightarrow \quad c < 2{,}500$
So there is a **pooling** equilibrium

Seller

- 1 with **good** item offers item for sale at a high price **high price (p)**
- 1 with **bad** item offers item for sale at low price **high price (p)**

Buyer's belief

- 1 $\text{prob}(\text{node } g \mid p) = p(G)$
- 1 $\text{prob}(\text{node } b \mid q) = p(B)$

1 Given beliefs
1 **buyer's strategy:** buy whatever is offered for sale **as long as** $p(G) > p(B)$

13 Proof of Case 3: $c < 2{,}500$ and $EV_B(\text{buy at } p) > 0$

Start with buyer (last mover)

Buyer observes that for **seller**

- 1 If **seller** knows item is **good** then offers to sell at **high price p** since
  $V_S(\text{sale at } p \mid G) = 3{,}000 > V_S(\text{sale at } q \mid G) = 500$
  $\Rightarrow$ Offer **good** item for sale at **high price p** with $\text{prob}(p \mid G) = 1$
  So offer **good** item for a **low price q** with $\text{prob}(q \mid G) = 1 - p(p \mid G) = 1 - 1 = 0$

- 1 If **seller** knows item is **bad** and willing to incur fix up costs of **c<2,500**, since

$V_S(\text{sale at p} \mid B) = 3{,}000 - c > V_S(\text{sale at q} \mid B) = 500$

$\Rightarrow$ Offer item for sale at **high price p** with $\text{prob}(p \mid B) = 1$

So offer **bad** item for a **low price q** with $\text{prob}(q \mid B) = 1 - p(p \mid B) = 1 - 1 = 0$

Buyer then knows that

- probability **seller** offers item at a **high price p** is the **marginal probability of a high price p**

$$\text{prob}(p) = p(G)p(p \mid G) + p(B)p(p \mid B)$$

1 $$= p(G) \times 1 + p(B) \times 1 = p(G) + p(B)$$

$$= 1$$

Therefore, **buyer's belief** that item is **good** given that it is offered at a **high price p**

$$\text{prob}(G \mid p) = \frac{p(G)p(p \mid G)}{p(p)} = \frac{p(G) \times 1}{1} = p(G)$$

1

1/2 $\Rightarrow$ observing a **high price p, buyer believes** in **node g** with prob $\text{prob}(G \mid g) = p(G)$

1/2 and **buyer's belief** that item is **bad** given that it is offered at a **high price p**

$$\text{prob}(B \mid p) = \frac{p(B)p(p \mid B)}{p(p)} = \frac{p(B) \times 1}{1} = p(B)$$

1/2 $\Rightarrow$ observing a **high price p, buyer believes** in **node b** with prob $\text{prob}(B \mid p) = p(B)$

(Note that since **seller adopt the same pricing strategy**, price is uninformative as to quality of the item and so the **buyer's belief** of being in **node g** is the same as the probability that **natures picks a good item**, and the **buyer's belief** of being in **node b** when there is a high price is the same as the **probability that nature picks a bad item**.)

Note that the probability that seller offers item at a low price q is

$$\text{prob}(q) = p(G)p(q \mid G) + p(B)p(q \mid B)$$

1/2 $$= p(G) \times 0 + p(B) \times 0$$

$$= 0$$

1 $\Rightarrow$ **Buyer's belief seller** will **never** charge a **low price q regardless** of quality of item

Given **buyer's beliefs**

- **1/2** if **buyer buys good** item offered at **high price (p)**
  her payoff if item is good is $V_B(\text{buy at p} \mid \text{node g}) = 2{,}000$
- **1/2** if buyer buys **bad** item offered at **high price (p)**
  her payoff if item is **bad** is $V_B(\text{buy at p} \mid \text{node b}) = -2{,}000$

$\Rightarrow$ **Buyer believes** her **expected** payoff from buying the item **high price (p)** is

1/2 $$EV_B(\text{buy at p}) = p(G \mid p) \times 2{,}000 + p(B \mid p) \times [-2{,}000]$$

$$= 2{,}000[p(G) - p(B)]$$

1 $\Rightarrow$ Given bel.ief,

Buyer buys item **iff** $EV_B(\text{buy at p}) = 2{,}000[p(G) - p(B)] > 0$

1/2 That is when $p(G) > p(B)$, i.e., when **nature** delivers a **good item** with a higher probability than a **bad item**.

Note also that since $p(G) = 1 - p(B)$ **then**

$$p(G) > p(B)$$
$$1 - p(B) > p(B)$$
$$1 > 2p(B)$$
$$\frac{1}{2} > p(B) \text{ or } p(G) > \frac{1}{2}$$

So that **buyer buys** item only when **probability that nature** delivers a **bad item is less than a half** or equivalently, that **nature delivers** a **good item with probability greater than one half**.

Move back to seller (second mover)

Given **buyer's BR function**, if $p(G) > p(B)$

For **seller**

- **1** with **good**

  $V_S(\text{sale at } p \mid G) = 3{,}000 > V_S(\text{sale at } q \mid G) = 500$

$\Rightarrow$ **BR of seller** with the **good** item is to offer it for sale at a **high price p**

- **1** with **bad**

  $V_S(\text{sale at } p \mid B) = 3{,}000 - c > V_S(\text{sale at } q \mid B) = 500$

$\Rightarrow$ **BR of seller** with the **bad** item is to offer it for sale at a **high price p**

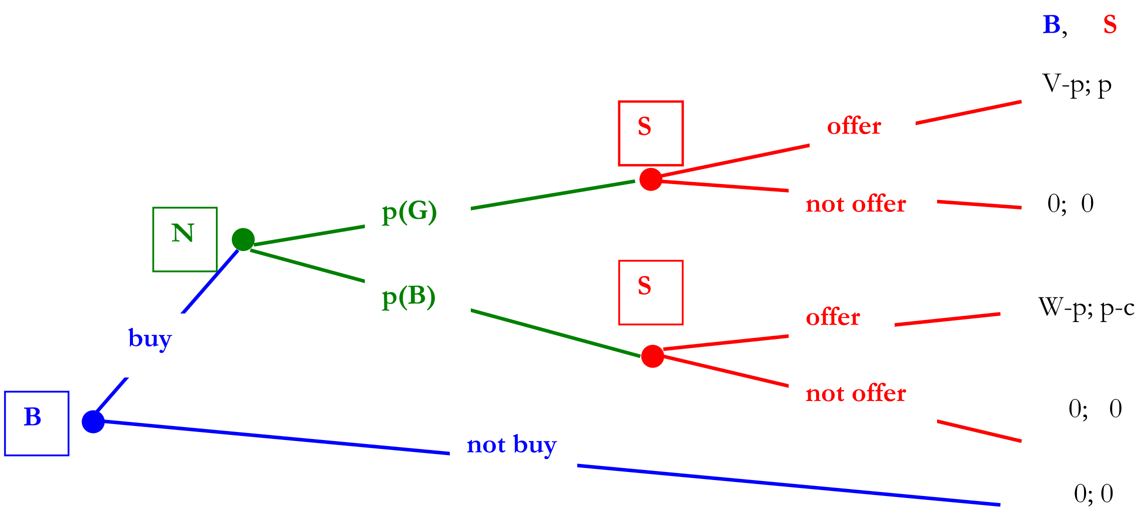
End of proof of Case 3

1 $\Rightarrow$ **In Case 3**, we have a **pooling** equilibrium and **partial** market success.

Q4 Screening Games (NOT INCLUDED IN ASSIGNMENT # 6 AS NOT COVERED IN CLASS WHEN ASSIGNMENT WAS DUE)

Consider the Caveat Emptor Game but with the roles reversed. **The buyer must first say whether she is willing to buy or not the item**. The seller hears the buyer's answer. If the buyer is willing to buy the item, the seller has to say yes or no to selling the item **conditional** on whether the item is good or bad. The seller of a bad item still has to incur the cleanup costs, $c > 0$, or there is no deal. This becomes a **screening game**, since the **buyer is uninformed**.

(a) Draw the game tree showing the payoffs to the buyer and the seller.



(b) Using conditional probability arguments determine the range of values for $c > 0$ that give rise to complete and partial market success and complete market failure. Notice that the Near market failure regime does not exist in this situation.

Case 1: $c > p$

Seller finds it too costly to incur clean-up costs to mimic good item

Candidate for a sequential equilibrium
Buyer's belief that

- probability that item is **good quality** given that it is offered for sale $\text{prob}(G \mid O) = 1$
- probability that item is **bad quality** given that it is offered for sale $\text{prob}(B \mid O) = 0$

Given beliefs buyer's strategy: buy whatever is offered for sale

Given **buyer's strategy**
Seller

- with **good (high) quality item** offers item for sale
- with **bad (low) quality item** does **NOT** offer item for sale

Proof of Case 1:
Start with seller (last mover)

Seller observes whether item is **good** or **bad**

- If **seller** knows item is **good quality seller** wants to sell item at p

  $$EV_S(O \mid G) = p > EV_S(\text{no sale}) = 0$$

  $\Rightarrow$ **seller** offer item for sale with $\text{prob}(O \mid G) = 1$

- If **seller** knows item is **bad quality** and must incur **fix up costs of c>p**
  Leading to loss of

  $$EV_S(O \mid B) = p - c < EV_S(\text{no sale}) = 0$$

  $\Rightarrow$ **seller** offers item for sale with $\text{prob}(O \mid B) = 0$, that is, **seller does not offer** it for sale

Nature (second mover) has no strategies to choose so **move back to buyer**

Move back to Buyer (first mover)

Buyer's belief on whether item will be offered for sale

$$\begin{aligned} p(O) &= p(G)p(O|G) + p(B)prob(O|B) \\ &= p(G)\times 1 + p(B)\times 0 \\ &= p(G) \end{aligned}$$

Therefore **buyer's belief** that

- **good quality item** is offered for sale is

$$p(G|O) = \frac{p(G)\times p(O|B)}{p(O)} = \frac{p(G)\times 1}{p(G)} = 1$$

- **bad quality item** is offered for sale is

$$p(B|O) = \frac{p(B)\times p(O|B)}{p(O)} = \frac{p(B)\times 0}{p(G)} = 0$$

Consequently, **Buyer** then **believes** that with probability 1
seller offers item for sale **only** when item is of good quality

Buyer's belief on quality of item leads her to believe that her **expected payoff from buying** is

$$\begin{aligned} EV_B(buy) &= p(G|O)\times[V-p] + p(B|O)\times[W-p] \\ &= 1\times[V-p] + 0\times[W-p] = V-p \end{aligned}$$

and since $EV_B(no\,buy) = p(G|O)\times 0 + p(B|O)\times 0 = 0$

$\Rightarrow \quad EV_B(buy) = V-p > EV_B(no\,buy) = 0$

So the **buyer's BR** is to commit to buying good **ahead of time**.

End of proof of Case 1

$\Rightarrow$ **In Case 1**, we have a **separating equilibrium and complete market success**

Case 2: $c < p$

Seller finds it cheap to clean-up item to **mimic** good item

Candidate for Sequential Equilibrium

Seller with **bad item** wants to mimic **good** item as **not too expensive** to do so.

Candidate for a sequential equilibrium

Buyer's belief

$$prob(G|O) = p(G) \text{ and } prob(B|O) = p(B)$$

Given beliefs buyer's strategy: buy whatever is offered for sale if $p(B)$ is **low enough**

Seller with

- **good (high) quality** item offers item for sale
- **bad (low) quality** item offers item for sale

Proof:

Start with **seller** (**last** mover)

Seller observes whether item is good or bad

- If **seller** knows item is **good quality** **seller** wants to sell item at p since

  $$EV_S(O|G) = p > EV_S(\text{no sale}) = 0$$

  $\Rightarrow$ **seller** offers item for sale at p with $prob(O|G) = 1$

- If **seller** knows item is **bad quality** and must incur **fix up costs of c<p** then

  $$EV_S(O|B) = p - c > EV_S(\text{no sale}) = 0$$

  $\Rightarrow$ **seller** offers item for sale at p with $prob(O|B) = 1$

Nature (second mover) has no strategies to choose so move back to buyer

Move back to **Buyer**

Buyer's belief on probability that item will be offered for sale is

$$\begin{aligned} p(O) &= p(G)p(O|G) + p(B)prob(O|B) \\ &= p(G)\times 1 + p(B)\times 1 \\ &= p(G) + p(B) \\ &= 1 \end{aligned}$$

Therefore the **buyer's belief** that

- **good quality item** if offered for sale is

  $$p(G|O) = \frac{p(G)\times p(O|G)}{p(O)} = \frac{p(G)\times 1}{1} = p(G)$$

- **bad quality item** if offered for sale is

  $$p(B|O) = \frac{p(B)\times p(O|B)}{p(O)} = \frac{p(B)\times 1}{1} = p(B)$$

Consequently, **Buyer** then **believes** that both **good** and **bad** quality items will be offered for sale so that probabilities that **good** and **bad** item is offered for sale is just is just the respective probabilities that **nature** picks a **good** and a **bad** item

Buyer's belief on quality of item leads her to believe that her **expected payoff from buying** is

$$\begin{aligned} EV_B(buy) &= p(G|O)\times[V-p] + p(B|O)\times[W-p] \\ &= p(G)\times[V-p] + p(B)\times[W-p] \end{aligned}$$

The **buyer's expected payoff** from not buying the item is then

$$EV_B(buy) = p(G|O)\times 0 + p(B|O)\times 0 = 0$$

So that **Buyer buys item** only if

$$EV_B(buy) = p(G)\times[V-p] + p(B)\times[W-p] > EV_B(\text{no buy}) = 0$$

Buyer indifferent from buying and not buying when

$$p(G)\times[V-p] + p(B)\times[W-p] = 0$$

$$[1-p(B)]\times[V-p] + p(B)\times[W-p] = 0$$

$$V - p - p(B) \times [V - p - W + p] = 0$$
$$V - p - p(B) \times [V - W] = 0$$
$$\frac{V - p}{V - W} = p(B)$$

Therefore, we have that

- if $\frac{V-p}{V-W} > p(B)$ so that $EV_B(buy) > 0$ then buyer buys item
  Pooling equilibrium and partial market success
- if $\frac{V-p}{V-W} < p(B)$ then $EV_B(buy) < 0$ buyer does NOT buy item
  Pooling equilibrium and complete market failure

(c) In a graph show the three market regimes.