Solutions for Final Exam 2012

Multiple Choice: 1. I 2. V 3. I 4. E 5. F 6. P 7. Q 8. N

Short Answer:

1. Worker: Cost = 40 – 40(1 – 0.02X) = 0.8X Benefit = 2[(40 + 3X) – 40] = 6X

If Benefit > Cost → 6X – 0.8X > 0 → 5.2X > 0 → X > 0

Firm: Cost = 40(1 – 0.02X) – 10 = 30 – 0.8X Benefit = 2[60 – (40 + 3X)] = 40 – 6X

If Benefit > Cost → (40 – 6X) – (30 – 0.8X) > 0 → 10 > 5.2X → 1.923 > X

The period one wage range is: 40[1 – 0.02(1.923)] = $38.46 to $40

2. Let X represent the amount Jake gets paid in periods 1-3, and (600-1.5X) in periods 4 and 5; here's the table:

Period	No shirk	Shirk	Net
1	X+X+X+(600 – 1.5X) +(600 – 1.5X) = 1200	202 + 0.75X + 0.25(160) + 4(160) = 882 + 0.75X	1200 – (882 + 0.75X) > 0 →318 > 0.75X → 424 > X
2	X+X+(600 – 1.5X) +(600 – 1.5X) = 1200 – X	302 + 0.75X + 0.25(160) + 3(160) = 822 + 0.75X	(1200 – X) – (822 + 0.75X) > 0 →378 > 1.75X → 216 > X
3	X+(600 – 1.5X) +(600 – 1.5X) = 1200 – 2X	290 + 0.75X + 0.25(160) + 2(160) = 650 + 0.75X	(1200 – 2X) – (650 + 0.75X) > 0 →550 > 2.75X → 200 > X
4	(600 – 1.5X) +(600 – 1.5X) = 1200 – 3X	166 + 0.75(600 – 1.5X) + 0.25(160) + 160 = 816 – 1.125X	(1200 – 3X) – (816 – 1.125X) > 0 →384 > 1.875X → 204.8 > X
5	600 – 1.5X	32 + 0.75(600 – 1.5X) + 0.25(160) = 522 – 1.125X	(600 – 1.5X) – (522 – 1.125X) > 0 →78 > 0.375X → 208 > X

Thus, the minimum period 4 wage is: 200 > X → W > $300

3.(a) Optimal: (i) Efficient effort if properly defined contract
(ii) Selection of high-productivity workers

Negative: (i) Uncertainty in production (observable & risk averse) → labour intensive
(ii) Uncertainty in production (unobservable & risk averse) → inefficient
(iii) Run down capital stock
(iv) Quantity over quality
(v) Costly to measure output

(b) Optimal: (i) Observable uncertainty (RA): easier to run, since no need to adjust contract
(ii) Unobservable uncertainty (RA): can obtain efficient effort

Negative: (i) Sabotage
(ii) Heterogeneous ability
(iii) collusion

(c) (i) Merit: Simple pay system if Q/N. If given equal share through VP, then optimal E
Drawback: Inefficient if Q/N → the 1/N problem

(ii) Merit: Can get to efficient target – a discontinuous reward schedule i.e. forcing contract
Drawback: coordination problem

(iii) Merit: Works well to get efficient
Drawback: expensive

4(a) U-max:

Worker #1: $U = 0.25(E_1 + E_2 + \ldots + E_6) - E_1^2/4 \rightarrow \delta U/\delta E_1 = 0.25 - 0.5E_1 = 0 \rightarrow E_1 = 0.5$

Worker #2: $U = 0.15(E_1 + E_2 + \ldots + E_6) - E_2^2/4 \rightarrow \delta U/\delta E_2 = 0.15 - 0.5E_2 = 0 \rightarrow E_2 = 0.3$

$Q = 0.5 + 5*0.3 = 2$, so $Y_1 = 0.5$ and $U_1 = 0.5 - (0.5^2)/4 = 0.4375$, and

$Y_2 = 0.3$ and $U_1 = 0.3 - (0.3^2)/4 = 0.2775$

Socially efficient: $\Sigma U = Q - \Sigma C(E) \rightarrow \delta\Sigma U/\delta E_1 = 1 - 0.5E_1 = 0 \rightarrow E_1 = 2$

$\rightarrow \delta\Sigma U/\delta E_2 = 1 - 0.5E_1 = 0 \rightarrow E_1 = 2$

$Q = 6*2 = 12$, so $Y_1 = 3$ and $U_1 = 3 - (3^2)/4 = 0.75$, and $Y_2 = 1.8$ and $U_2 = 1.8 - (1.8^2)/4 = 0.99$, and

(b) Let pay schedule for worker #1 be $Y_1 = a_1 + bQ \rightarrow U_1 = (a_1 + bQ) - E_1^2/4$

$\rightarrow \delta U/\delta E_1 = b - E_1/2 = 0 \rightarrow 2b = E_1 \rightarrow b=1$

Let pay schedule for worker #2 be $Y_2 = a_2 + bQ \rightarrow U_2 = (a_2 + bQ) - E_2^2/4$

$\rightarrow \delta U/\delta E_2 = b - E_2/2 = 0 \rightarrow 2b = E_2 \rightarrow b=1$

Then, $Y_1 = 0.25Q \rightarrow (a_1 + bQ) = 0.25Q \rightarrow (a_1 + (1)Q) = 0.25Q \rightarrow 0.75Q = -a_1 \rightarrow -9 = a_1$

And, $Y_2 = 0.15Q \rightarrow (a_2 + bQ) = 0.15Q \rightarrow (a_2 + (1)Q) = 0.15Q \rightarrow 0.85Q = -a_2 \rightarrow -10.2 = a_2$

5(a) Pareto optimal effort:

$\text{Max}(E(\Pi) + E(U_1) + E(U_2)) = [E(Q) - E(Y_1) - E(Y_2)] + [E(Y_1) - C(E_1)] + [E(Y_2) - C(E_2)]$

$= E(Q) - C(E_1) - C(E_2)$

$= 4E_1 + 4E_2 - E_1^2 - E_2^2$

$\rightarrow \delta(E(\Pi) + E(U_1) + E(U_2))/\delta E_1 = 4 - 2E_1 = 0 \rightarrow E_1 = 2$

$\rightarrow \delta(E(\Pi) + E(U_1) + E(U_2))/\delta E_2 = 4 - 2E_2 = 0 \rightarrow E_2 = 2$

$E(U_1) = a + P_1S - E_1^2 = a + [0.5 + 0.25(E_1 - E_2)]S - E_1^2$

$\rightarrow \delta(E(U_1))/\delta E_1 = 0.25S - 2E_1 = 0 \rightarrow S = 2E_1/0.25 = 16$

If $E(U_1) = 1 \rightarrow 1 = a + P_1S - E_1^2 \rightarrow 1 = a + (0.5)(16) - (2)^2 \rightarrow a = -3$

So $E(q_1) = 4(2) = 8$, $E(Y) = -3 + (0.5)16 = 5$, $E(U) = 1$, and $E(\Pi) = 8 - 5 = 3$.

(b) From prior question we know that the Pareto optimal effort is E = 2 for both workers.

Maximizing utility:
$E(U_1) = a + P_1S - E_1^2 = a + [0.5 + (4/2X)(E_1 - E_2)]S - E_1^2$
$\rightarrow \delta(E(U_1))/\delta E_1 = (2/X)S - 2E_1 = 0 \rightarrow S = XE_1 = 2X$

If $E(U_1) = 1 \rightarrow 1 = a + P_1S - E_1^2 \rightarrow 1 = a + (0.5)(2X) - (2)^2 \rightarrow a = 5 - X$

If $a > 0 \rightarrow 5 - X > 0 \rightarrow X < 5$

6. The key issue here is to determine the range in education for each job that will separate the three workers appropriately. Given that there is no educational requirement for Job #1, we can assume that utility-maximizing workers will set education equal to zero to take this job. However, we'll denote the educational requirement for job #2 as "E_2", and the educational requirement for job #3 as "E_3". Using this notation, we note that less-productive workers must prefer job #1 to job #2:
$U_L(\text{job \#1}) > U_L(\text{job \#2}) \rightarrow 2 > 6 - (1.25)E_2 \rightarrow E_2 > 3.2$

Also, medium-productive workers must prefer Job #2 to job #1:
$U_M(\text{job \#2}) > U_M(\text{job \#1}) \rightarrow 6 - [0.4 + (0.5)E_2] > 2 - [0.4 + (0.5)0] \rightarrow 4 > (0.5)E_2 \rightarrow 8 > E_2$

Thus, in order to separate workers between jobs 1 and 2, it is necessary for the educational requirement at job #2 to range as: $3.2 < E_2 < 8$

To determine the appropriate range for E_3, it is necessary to induce high-productive workers to prefer job #3 to job #1:
$U_H(\text{job \#3}) > U_H(\text{job \#1}) \rightarrow 10 - (0.2)E_3 > 2 \rightarrow 8 > (0.2)E_3 \rightarrow 40 > E_3$
This information will also be useful later, so I'll denote "equation (1)" as: $40 > E_3$

Also, it is necessary that these workers prefer job #3 to job #2:
$U_H(\text{job \#3}) > U_H(\text{job \#1}) \rightarrow 10 - (0.2)E_3 > 6 - (0.2)E_2 \rightarrow 4 > (0.2)E_3 - (0.2)E_2 \rightarrow 20 > E_3 - E_2$
This information will be useful later, so I'll denote "equation (2)" as: $20 > E_3 - E_2$

Also, it is necessary that medium-productive workers prefer job #2 to job #3:
$U_M(\text{job \#2}) > U_M(\text{job \#3}) \rightarrow 6 - [0.4 + (0.5)E_2] > 10 - [0.4 + (0.5)E_3] \rightarrow (0.5)E_3 - (0.5)E_2 > 4 \rightarrow E_3 - E_2 > 8$
This information will also be useful later, so I'll denote "equation (3)" as: $E_3 - E_2 > 8$

Given that $3.2 < E_2 < 8$, it is possible to back out a range for E_3 that satisfies both equation 1 and 2. That is: (a) if $E_2 = 3.2$, then equation (1) implies that $E_3 < 40$, and equation (2) implies that $E_3 < 23.2$
(b) if $E_2 = 8$, then equation (1) implies that $E_3 < 32$, and equation (3) implies that $E_3 > 16$

Therefore, it is necessary that $23.2 > E_3 > 16$ in order to satisfy both (a) and (b) (and it satisfies the requirement of equation (1), too).