Phys 131 Final - 2008

Note Title 12/18/2009

Solutions

Conceptual questions

1) At the poles.

Greatest difference: at equator (where

$F_N - mg = -mv^2/r$

$F_N = mg - \frac{mv^2}{r} < mg$.

2) The top book has kinetic energy that it did not previously have. So work has been done on it, by the force of (static) friction between it and the book underneath it.

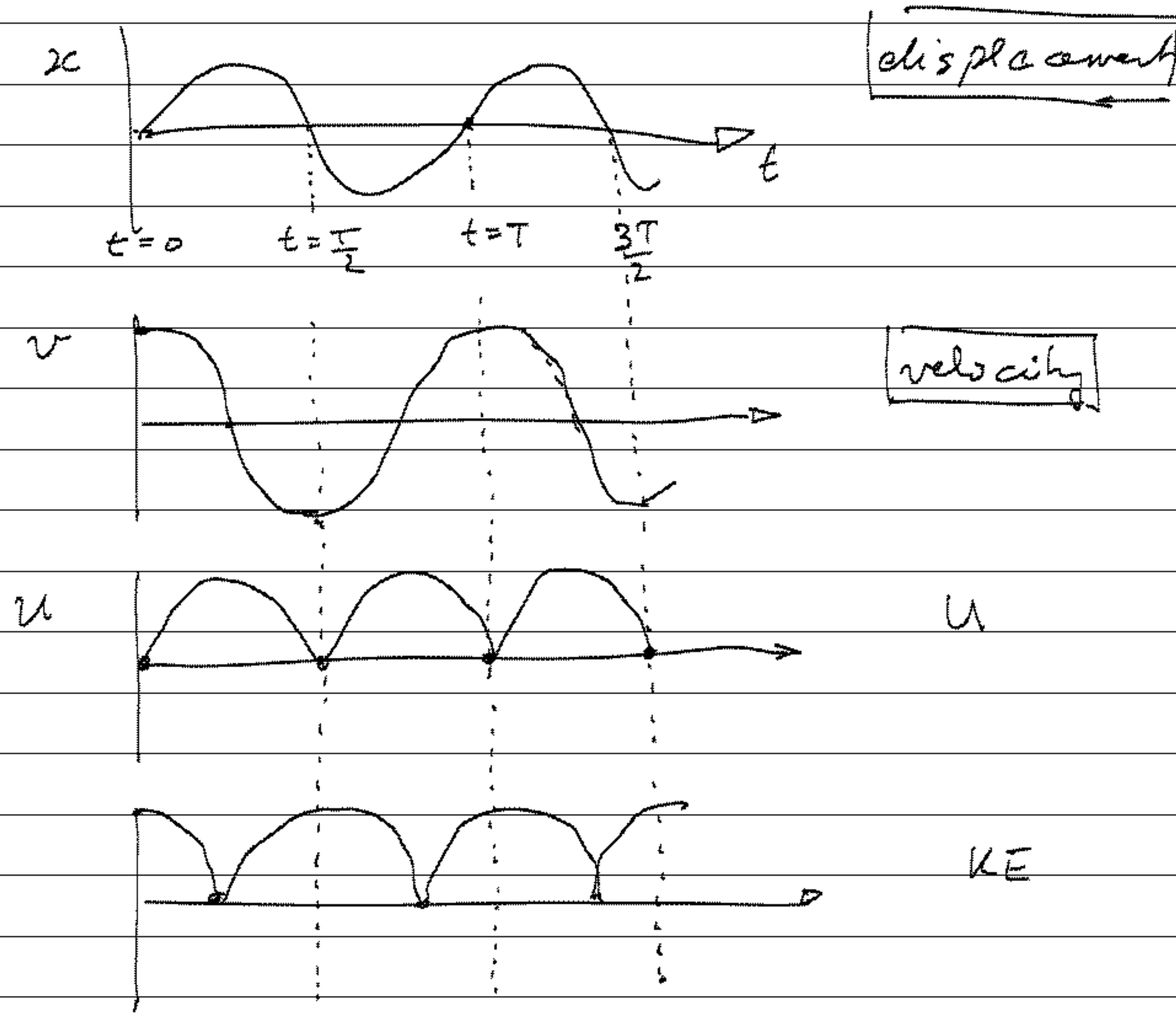
3) The ball is in projectile motion with an external force (gravity) acting on it. Thus, $\vec{p}$ is not constant ... but $\vec{p}_x$ (horiz. component) is.

4) Ice is at poles where $r \to 0$ so contribution to I is small. As it spreads out over ocean, it contribution to I increases, so I_{earth} increases. Since angular mom. $L = I\omega$ is conserved (no external torques), ω will decrease and T (period = length of day) will increase slightly.

5) No acceleration $\Rightarrow F_{net} = 0$ so
$\vec{F}_1 = -\vec{F}_2$.

No torque, so forces must act through same point.

6)



7) $v = \lambda f$ $\quad v = \sqrt{F_T/\mu}$ $\quad \mu$ = mass/length

a) λ depends directly on length only.
(ie if you change F_T or μ, λ of fundamental doesn't change)

b) λ of sound in air $= \dfrac{v_{sound}}{f} = \dfrac{\lambda_{string}\, v_{sound}}{\sqrt{F_T/\mu}}$

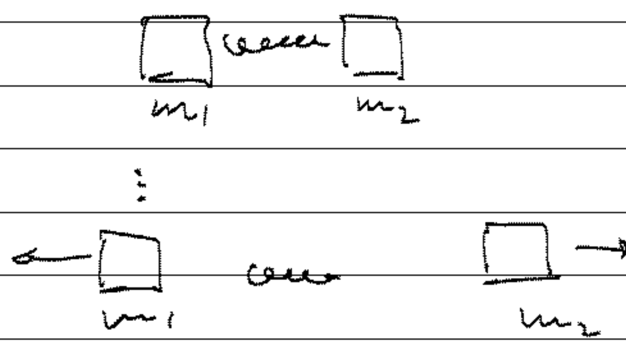
Dependence on F_T, λ_{string}, and μ.

8) $F_x = -dU/dx$

So F is related to gradient of force, not force itself.
So $F = 0$ does not imply $U = 0$

Similarly, $U = 0$ does not imply $F = 0$.

9) All forces are internal



So $\Delta \bar{p} = 0$ so blocks have equal momenta.

10) $y = A \sin(Bx - Ct)$ $\quad A, B, C > 0$

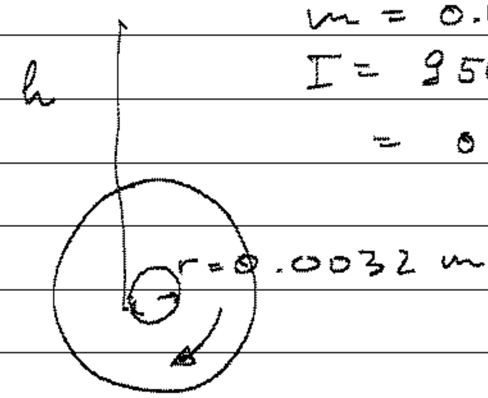
a) Wave propagates in +ve x direction
[general form: $y = A \sin(kx - \omega t)$

b) $k = 2\pi/\lambda = B \implies \boxed{\lambda = \frac{2\pi}{B}}$.

c) $\boxed{f = 2\pi\omega = 2\pi C}$

Long questions.

1 a)



$m = 0.120$ kg

$I = 950 \text{ gm cm}^2$

$= 0.950 \text{ kg cm}^2 = 9.50 \cdot 10^{-5} \text{ kg m}^2$

FBD: $\uparrow T$, $\downarrow mg$

take +ve down so $a > 0$

$mg - T = ma$

torque: $\tau = T \cdot r = I\alpha = I \frac{a}{r}$ $\quad (a = \alpha r)$

At bottom (just before end of string), we can think about energy:

$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \qquad v = \omega r$

$= \frac{1}{2} mv^2 + \frac{1}{2} I(v/r)^2 = \frac{v^2}{2}\left(m + \frac{I}{r^2}\right)$

$v^2 = \dfrac{2mgh}{m + I/r^2} = 0.300 \text{ m}^2/\text{s}^2$

$v = 0.548$ m/s

So $a = v_f^2 / 2d = \dfrac{0.30^2}{2 \cdot 1.20} = \underline{0.125 \text{ m/s}^2}$

b) $d = \frac{1}{2} a t^2$

$t = 2d/a = 2 \cdot 1.20 \text{ m} / 0.125 \text{ m/s}^2$

$\boxed{t = 4.38 \text{ sec}}$

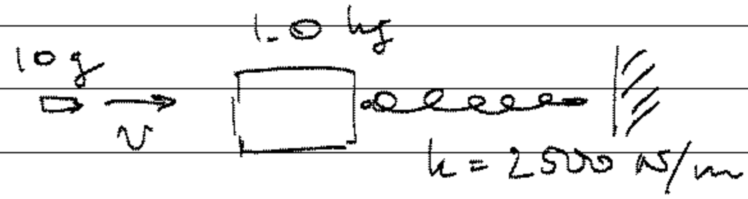
c) $v = 0.548$ m/s as calculated above

d) $\frac{1}{2} m v^2 = 0.0180 \text{ J}$

e) $v = \omega r \qquad \omega = v/r = \frac{0.548}{0.0032}$

$$\boxed{\omega = 171 \text{ rad/s}} \quad (= 27 \text{ rev/sec})$$

2



a) amplitude = 10.0 cm

So $E = \frac{1}{2} k A^2 = \frac{1}{2} \cdot 2500 \cdot (0.01)^2$

$= 0.125 \text{ J}$

$E = \frac{1}{2} m v_{max}^2$

Thus $v_{max} = \left(\frac{0.125 \text{ J}}{\frac{1}{2}(1.01 \text{ kg})} \right)^{1/2}$

$= 0.498 \text{ m/s}$

In collision, $\vec{p}$ is conserved:

$$m_{bullet} \cdot v_{bullet} = (m_{block} + m_{bullet})\, v_{max}$$

$$\boxed{v_{bullet} = 50.2 \text{ m/s}}$$

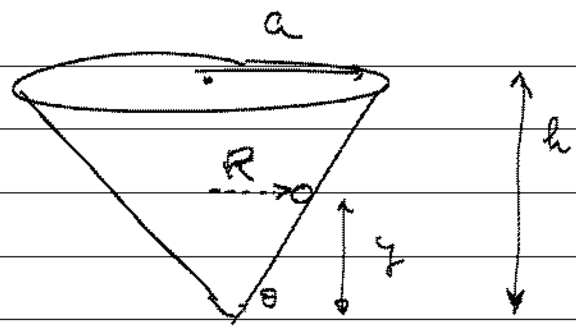
b) If told f $\left(= \frac{1}{2\pi}\sqrt{\frac{k}{m}}\right)$

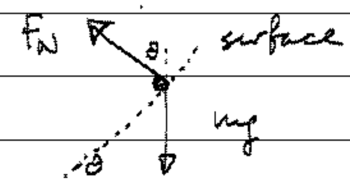
we only know the ratio of k/m. This is not enough information to calculate v_{max} and m, both of which we need to know to work back to v_{bullet}.

c) Here, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2500}{1.01}}$

$$\boxed{f = 7.92 \text{ Hz}}$$

(3)

a) 

FBD: 

vertical component of F_N = mg

$$F_N \cos\theta = mg$$

horiz. component of $F_N = \frac{mv^2}{R} = F_N \sin\theta$

So $$\frac{\cancel{m}v^2}{R} = \cancel{m}g \tan\theta$$

$$v^2 = Rg \tan\theta = \frac{Rgh}{a}$$

But $R/y = a/h$ so $R = ay/h$

$$v^2 = yg$$

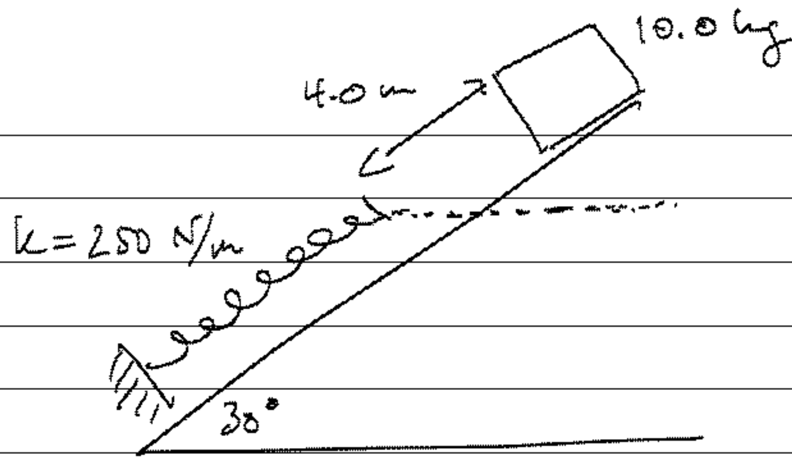
$$\boxed{v = \sqrt{yg}}$$

b) total KE $= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\quad v = \omega r$

$$= \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}m\cancel{r^2}\frac{v^2}{\cancel{r^2}}$$

$$\boxed{KE = \frac{7}{10}mv^2 = \frac{7}{10}yg\frac{h^2}{a^2}}$$

(4)



a) Max. compression of spring?
Take zero of height to be uncompressed pos^n.

$$mg\,(4.0 \times \sin 30°) = -mg(x \cdot \sin 30°) + \tfrac{1}{2}kx^2$$

where x = max. compression (+ve quantity)

$$\boxed{x = 1.46 \text{ m}}$$

b) Max speed occurs when acceleration changes sign; ie when $F = 0$

So $\quad mg \sin 30° = kx$

$$x = mg \sin 30° / k = 0.196 \text{ m}$$
(compression)

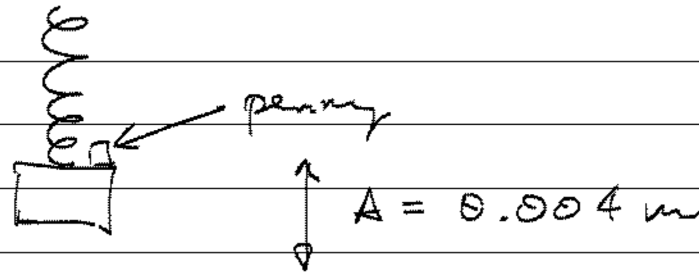
c) Initial acceleration:

$$F_{\text{up ramp}} = kx - mg \sin 30° = ma$$

$$a = \frac{kx}{m} - g \sin 30°$$

$$\boxed{a = 31.6 \text{ m/s}^2}$$

(5)



As f is increased, a_{max} increases.

a) penny first loses contact at highest point, where a_{max} is downwards and equal to or greater than g.

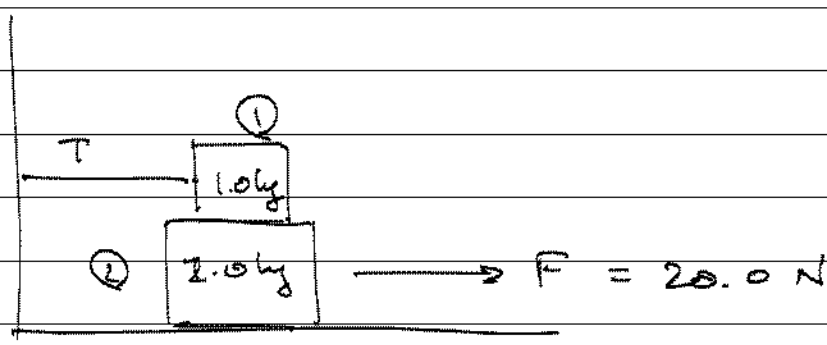
b) Penny just barely in contact means accel. of block downwards = g

$$a_{max} = \omega^2 A = g \qquad \omega = 2\pi f$$

$$f = \left(\frac{g}{A}\right)^{1/2} \frac{1}{2\pi}$$

$$\boxed{f = 7.88\ Hz}$$

(6)



$\mu_k = 0.40$ at upper & lower surfaces

a) FBD of block ① :

$\uparrow F_{N\,(2\ on\ 1)}$

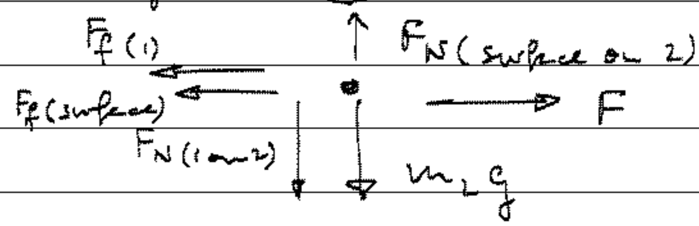
T $\quad$ $F_f = \mu_k \cdot F_N = \mu_k m_1 g$

$m_1 g$

so $\qquad T = \mu_k m_1 g \qquad = 0.40 \cdot 1.0 \cdot 9.8\ N$

$$\boxed{T = 3.92 \text{ N}}$$

b) FBD of block 2:



$$F_{N(\text{surface on } 2)} = m_2 g + F_{N(1 \text{ on } 2)}$$
$$= (m_1 + m_2) g$$

So

$$F_{f(\text{surface})} = \mu_k (m_1 + m_2) g$$
$$F_{f(1)} = \mu_k m_1 g$$

Thus

$$F_{net} = F - \mu_k (m_1 + m_2) g - \mu_k m_1 g$$
$$= F - \mu_k (2m_1 + m_2) g$$
$$= 20.0 \text{ N} - 0.40 (4.0 \text{ kg}) 9.8 \text{ m/s}^2$$
$$= 4.32 \text{ N}$$
$$F_{net} = ma$$

$$\boxed{a_2 = 4.32 \text{ N} / 2.0 \text{ kg} = 2.16 \text{ m/s}^2}$$