Answers to F2011 Exam

1. $\displaystyle\int_{\theta=0}^{\pi/4}\int_{r=0}^{2\sin\theta}(1-r^2)r\,dr\,d\theta+\int_{\theta=\pi/4}^{\pi/2}\int_{r=0}^{\csc\theta}(1-r^2)r\,dr\,d\theta$

2. 0

3. $\alpha-\dfrac{\alpha}{\sqrt{13}}$

4. $\displaystyle\iint_S(\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,dS=\oint_C\mathbf{F}\cdot d\mathbf{r}=18\pi$. (Here, C is the curve of intersection of $z=\sqrt{x^2+y^2}$ with the plane $z=3$, and is parameterized by $x=3\cos t$, $y=3\sin t$, $z=3$, $0\le t\le 2\pi$.)

5. $\dfrac{8}{7}$

6. (a) $y(t)=\dfrac{1}{4}\sin(2t)-\dfrac{1}{4}\cos(2t)+\dfrac{1}{4}-\dfrac{1}{2}t$

   (b) $y=c_1\cos x+c_2\sin x+\cot x\cos x-\dfrac{1}{2\sin x}$

7. $\displaystyle\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\sin(nx)$

8. (a) $\phi(x,y)=x^2y^2-3x+4y$

   (b) -22

9. π. (Hint: Let the origin be at the centre of the larger sphere, and place the centre of the smaller sphere on the z-axis, at the point $(0,0,a)$. Now write down equations for the two spheres. To find the region in the xy-plane to integrate over, you'll need to determine the curve of intersection of the two spheres and project it into the xy-plane.)