DEPARTMENT OF MATHEMATICS

Final Exam

MTH 312 – Vector Calculus

Last Name (Print):________________. First Name:________________. Student Number: ____________

Signature:________________.

Date: Dec. 12, 2011, 8:00 am

Duration: 3 hours

Version **A**

Section(circle one)

C. Kim	1 2 3
J. Pascal	4 5 6 7

Instructions:

1. This is a closed-book test. **Notes, calculators and other aids are not permitted.**
2. Verify that your test has pages 1-9.
3. (a) Unless otherwise instructed, **make sure you include all significant steps in your solution, presented in the correct order. Unjustified answers will be given little or no credit. Cross out or erase all rough work not relevant to your solution.** Put a box around your final **answer.**

   (b) Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there. Marks (out of 100) are shown in brackets.
4. Do not separate the sheets.
5. Have your student card available on your desk.

For Instructor's use only.

Page	Mark
2	/16
3	/12
4	/12
5	/12
6	/12
7	/12
8	/12
9	/12
Total	/100

1. [8 marks] Rewrite the iterated integral $\int_0^1 \int_0^{\sqrt{2y-y^2}} (1 - x^2 - y^2)dxdy$ in polar coordinate form. **Do not evaluate the integral.**

2. [8 marks] Evaluate $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2)dxdy$ by reversing the order of integration.

3. [12 marks] Two electrical charges of like polarity (i.e., both positive or both negative) will repel each other. If a charge of α coulombs is placed at the origin and a charge of 1 coulomb of the same polarity is at the point (x, y), then the force of repulsion is given by $\mathbf{F}(x, y) = \dfrac{\alpha x}{(x^2+y^2)^{\frac{3}{2}}}\mathbf{i} + \dfrac{\alpha y}{(x^2+y^2)^{\frac{3}{2}}}\mathbf{j}$. How much work is done by the force on the 1-coulomb charge as the charge moves on the straight line from $(1, 0)$ to $(3, -2)$?

4. [12 marks] Verify Stokes' Theorem for $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j} - xyz\mathbf{k}$ where S is the part of the cone $z = \sqrt{x^2 + y^2}$ for $0 \leq x^2 + y^2 \leq 9$ oriented upward. You must use this specific surface when calculating the surface integral.

5. [12 marks] Use the divergence theorem to find $\int\int_S (\mathbf{F} \cdot \mathbf{n}) dS$, where $\mathbf{F}(x, y, z) = z^2\mathbf{i} + (y^2 + \sin x)\mathbf{j} + xy^3\mathbf{k}$ and S is the surface of the region bounded by the parabolic cylinder $y = x^2$ and the planes $z = y$, $y = 1$ and $z = 0$, oriented outward.

6. [12 marks]

(a) Use the Laplace transform to solve $y'' + 4y = 1 - 2t, \;\; y(0) = y'(0) = 0$.

(b) Use variation of parameters to solve the differential equation $\dfrac{d^2y}{dx^2} + y = \csc^3 x$.

Note: The general solution to the associated homogeneous equation is $y_c = c_1 \cos x + c_2 \sin x$.

7. [12 marks] Expand $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 \leq x < \pi. \end{cases}$ in a Fourier series.

8. [12 marks] Find a potential function ϕ for $\mathbf{F}(x,y) = (2y^2x-3)\mathbf{i}+(2yx^2+4)\mathbf{j}$, and use it to evaluated $\int_C \mathbf{F}\cdot\mathbf{dr}$ where C is a path from $(2,-3)$ to $(3,-1)$.

9. [12 marks] A sphere of radius 1 has its center on the surface of a sphere with radius $a > 1$. Find the surface area of that portion of the larger sphere cut out by the smaller sphere.